Hint-Based SMT Proof Reconstruction

Joshua Clune[1] Haniel Barbosa[2] Jeremy Avigad[1]

[1]Carnegie Mellon University, [2]Universidade Federal de Minas Gerais

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Established Proof Reconstruction Strategies

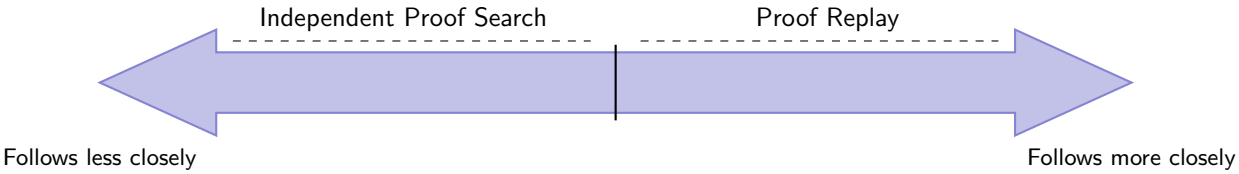
- Independent proof search on minimized facts
  - Used e.g. for superposition theorem provers and SMT solvers in Sledgehammer (Isabelle/HOL)
  - Used e.g. for superposition theorem provers in LeanHammer (Lean)
  - Used e.g. for superposition theorem provers and SMT solvers in CoqHammer (Rocq)

- Proof replay
  - Used e.g. for z3, veriT, and cvc5 in Sledgehammer (Isabelle/HOL)
  - Used e.g. for cvc5 in smt-lean (Lean)
  - Used e.g. for veriT in SMTCoq (Rocq)

- "The difference between walking and running"

Tradeoffs

These strategies differ in how closely they follow the original solvers' proofs.



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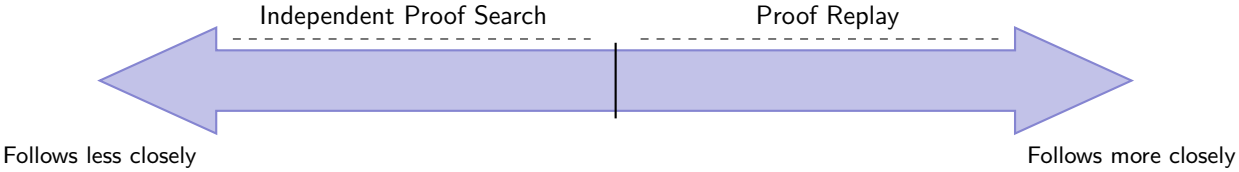

Following proofs less closely:

- ▷ Requires ITP automation to rediscover more
- ▷ Makes it possible to drop dependencies on external solvers

Following proofs more closely:

- ▷ Puts less burden on the ITP automation
- ▷ Requires retaining a dependence on the external solvers

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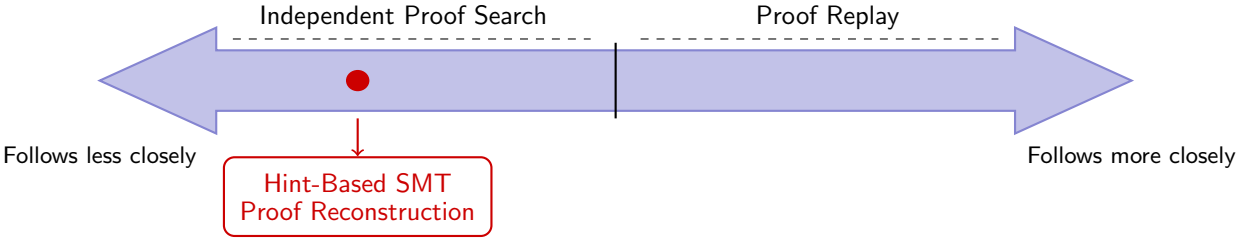

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Hint-Based SMT Proof Reconstruction

- SMT proofs consist both of `theory content` and `logical content`.
  - Theory content refers to reasoning on arithmetic, data structures, and so on.
  - Logical content refers to reasoning on connectives and quantifiers.

- Our reconstruction approach is to use hints output by the solver
  - reconstruct the theory content independently (e.g. via grind)
  - use independent proof search (duper) to reconstruct the logical content while `aided` by the theory content

- This is implemented as a Lean tactic called QuerySMT

What is QuerySMT?

QuerySMT is a tactic that:

- Translates Lean goals to SMT-LIB (using lean-auto)
- Receives hints on how to solve the goals from cvc5
- Translates those hints back to Lean
- Produces a standalone proof script (that doesn't depend on cvc5)

Example

```
example (f : ℤ → ℤ) (h1: ∀ x y, f x = f y → x = y)
  (h2 : ∃ x, ∀ y, f x ≤ f y) :
  ∃ x, ∀ (y : ℤ), x ≠ y → f x < f y :=by querySMT
```

Example

```
example (f : ℤ → ℤ) (h1: ∀ x y, f x = f y → x = y)
  (h2 : ∃ x, ∀ y, f x ≤ f y) :
  ∃ x, ∀ (y : ℤ), x ≠ y → f x < f y :=by querySMT
```

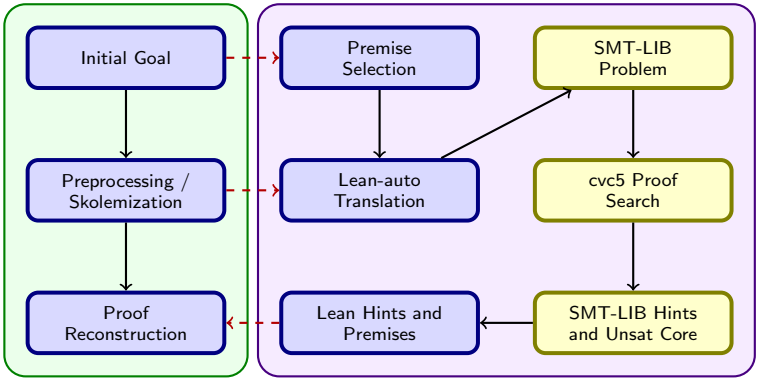
QuerySMT produces a standalone proof:

```
  @[classical] byContradiction
  intro negGoal
  skolemizeAll
  have smtLemma0 : ∀ (_i_0 : ℤ), ¬f sk0 ≤ -1 * f _i_0 ≥ 1 :=by grind
  have smtLemma1 : ∀ (bv0 : ℤ), f bv0 + -1 * f (sk1 bv0) ≥ 0 :=by grind
  have smtLemma2 : ∀ (bv0 : ℤ), ¬bv0 = sk1 bv0 :=by grind
  have smtLemma3 : (¬f sk0 + -1 * f (sk1 sk0) ≥ 0 ∨ f sk0 = f (sk1 sk0))∨
     f sk0 + -1 * f (sk1 sk0) ≥ 1 :=by grind
  duper [h1, smtLemma0, smtLemma1, smtLemma2, smtLemma3] []
```

QuerySMT Design

QuerySMT consists of five primary components:

1. Preprocessing
2. Translation
3. Hint generation
4. Hint interpretation
5. Proof reconstruction



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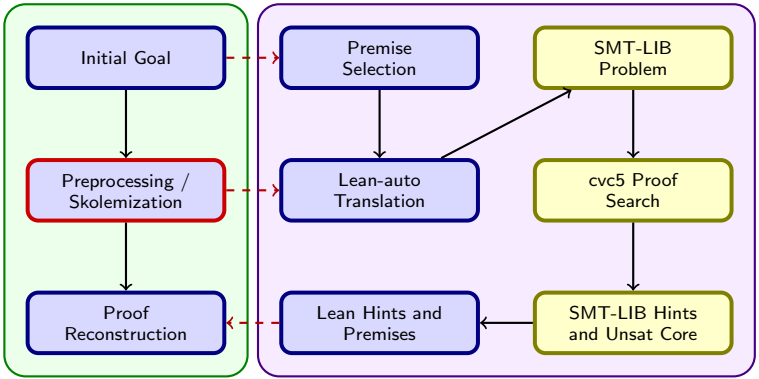

Preprocessing

```
example (f : ℤ → ℤ) (h1: ∀ x y, f x = f y → x = y)
(h2 : ∃ x, ∀ y, f x ≤ f y) :
∃ x, ∀ (y : ℤ), x ≠ y → f x < f y := by
```

▼ Tactic state

1 goal

```
f : ℤ → ℤ
h1 : ∀ (x y : ℤ),
f x = f y → x = y
h2 : ∃ x, ∀ (y :
ℤ), f x ≤ f y
⊢ ∃ x, ∀ (y : ℤ), x
≠ y → f x < f y
```

Preprocessing

```
example (f : ℤ → ℤ) (h1: ∀ x y, f x = f y → x = y)
(h2 : ∃ x, ∀ y, f x ≤ f y) :
∃ x, ∀ (y : ℤ), x ≠ y → f x < f y := by
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▼ Tactic state
1 goal

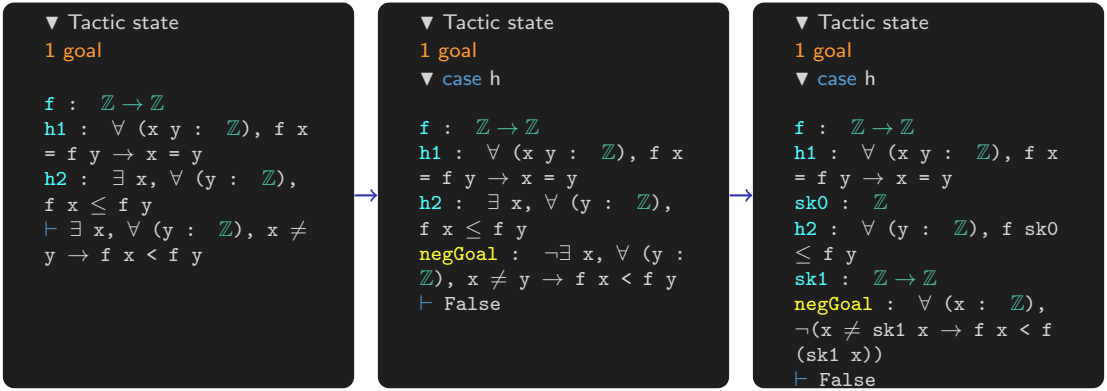
```
f : ℤ → ℤ
h1 : ∀ (x y : ℤ), f x
= f y → x = y
h2 : ∃ x, ∀ (y : ℤ),
f x ≤ f y
⊢ ∃ x, ∀ (y : ℤ), x ≠
y → f x < f y
```

▼ Tactic state
1 goal
▼ case h

```
f : ℤ → ℤ
h1 : ∀ (x y : ℤ), f x
= f y → x = y
h2 : ∃ x, ∀ (y : ℤ),
f x ≤ f y
negGoal : ¬∃ x, ∀ (y :
ℤ), x ≠ y → f x < f y
⊢ False
```

Preprocessing

```
example (f : ℤ → ℤ) (h1: ∀ x y, f x = f y → x = y)
(h2 : ∃ x, ∀ y, f x ≤ f y) :
∃ x, ∀ (y : ℤ), x ≠ y → f x < f y := by
@[classical] byContradiction
intro negGoal
skolemizeAll
```



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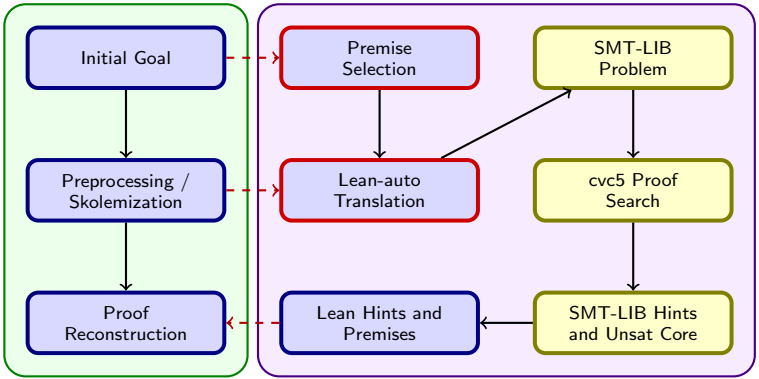

Translation

```
▼ Tactic state
1 goal
▼ case h

f : ℤ → ℤ, h1 : ∀ (x y : ℤ), f x = f y → x = y
sk0 : ℤ, h2 : ∀ (y : ℤ), f sk0 ≤ f y, sk1 : ℤ → ℤ
negGoal : ∀ (x : ℤ), ¬(x ≠ sk1 x → f x < f (sk1 x))
⊢ False
```

⇓ Lean-auto

SMT-LIB

```
(declare-sort Empty 0)
(define-fun nsub ((x Int) (y Int)) Int (ite (>= x y) (- x y) 0))
(define-fun itdiv ((x Int) (y Int)) Int (ite (>= y 0) x (ite (>= x 0) (div y (- (div (- x) y))))))
(define-fun itmod ((x Int) (y Int)) Int (ite (>= y 0) x (ite (>= x 0) (mod y (mod (- x) y))))))
(define-fun iediv ((x Int) (y Int)) Int (ite (>= y 0) (itdiv x y) (- (itdiv (- x) y))))
(declare-fun _sk1_ (Int) Int)
(declare-fun _f (Int) Int)
(assert (forall ((_i_0 Int)) (not (=> (not (= _i_0 (_sk1_ _i_0))) (< (_f _i_0) (_f (_sk1_ _i_0)))))) :named valid_fact_0)
(declare-fun _sk0_ () Int)
(assert (forall ((_i_0 Int)) (<= (_f _sk0_) (_f _i_0))) :named valid_fact_1)
(assert (forall ((_i_0 Int)) (not (=> (not (= _i_0 (_sk1_ _i_0))) (< (_f _i_0) (_f (_sk1_ _i_0)))))) :named valid_fact_2)
(assert (forall ((_i_0 Int) (_i_1 Int)) (=> (= (_f _i_0) (_f _i_1)) (= _i_0 _i_1))) :named valid_fact_3)
(assert (<= (- 5) (_f (_i_0 Int)) (_f (_i_1 Int)) (<= (_f _i_0) (- 5 (_f _i_1)))) :named valid_fact_4)
```

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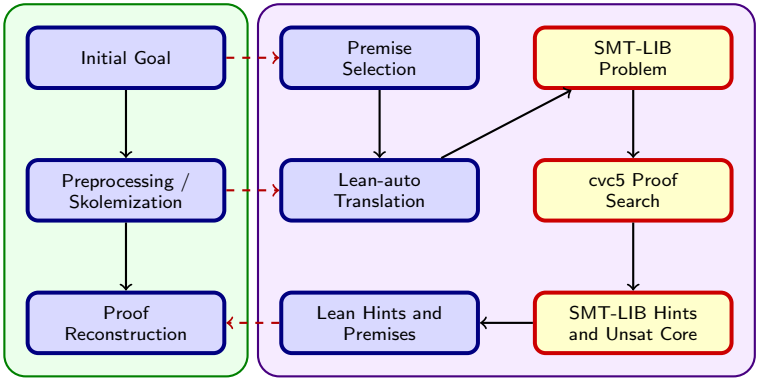

Hint Generation

- cvc5 is invoked with special flags:

```
cvc5 --tlimit=10000 --produce-models --enum-inst
--dump-hints --proof-granularity=dsl-rewrite
--hints-only-rw-insts --produce-proofs temp.smt2
```

Hint Generation

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cvc5 --tlimit=10000 --produce-models --enum-inst
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```

- cvc5 generates hints: preprocessing facts and theory lemmas

8 hints generated by cvc5

```
unsat
preprocessing facts:
(=> (forall ((_i_0 Int)) (<= (_f _sk0_) (_f _i_0))) (forall ((_i_0 Int)) (not (=> (+ (_f _sk0) (* (- 1) (_f _i_0))) 1))))
(=> (forall ((_i_0 Int)) (forall ((_i_2 Int)) (=> (= (_f _i_1) (_f _i_2)) (= _i1 _i_2))))
(forall ((_i_1 Int) (_i_2 Int)) (or (not (= _f _i_1) (_f _i_2)) (= _i_1 _i_2))))
(=> (forall ((_i Int)) (let ((_let_1 (_sk1_ _i))) (not (=> (not (= _i _let_1) (< (_f _1) (_f _let_1)))))))
(forall ((BOUND_VARIABLE_4960 Int)) (>= (+ (_f BOUND_VARIABLE_4960) (* (- 1) (_f (_sk1_ BOUND_VARIABLE_4960)))) 0)))
(=> (forall ((_i Int)) (let ((_let_1 (_sk1_ _i))) (not (=> (not (= _i _let_1)) (< (_f _i) (_f _let_1)))))))
(forall ((BOUND_VARIABLE_4953 Int)) (not (= BOUND_VARIABLE_4953 (_sk1_ BOUND_VARIABLE_4953)))))
theory lemmas:
(let ((_let_1 (_f (_sk1_ _sk0_)))) (let ((_let_2 (_f _sk0_)))
(let ((_let_3 (+ _let_2 (* (- 1) _let_1)))) (or (not (>= _let_3 0)) (= _let_2 _let_1) (>= _let_3 1)))))
(let ((_let_1 (+ (_f _sk0_) (* (- 1) (_f (_sk1_ _sk0_)))))) (=> (< _let_1 1) (<= _let_1 0)))
...
```

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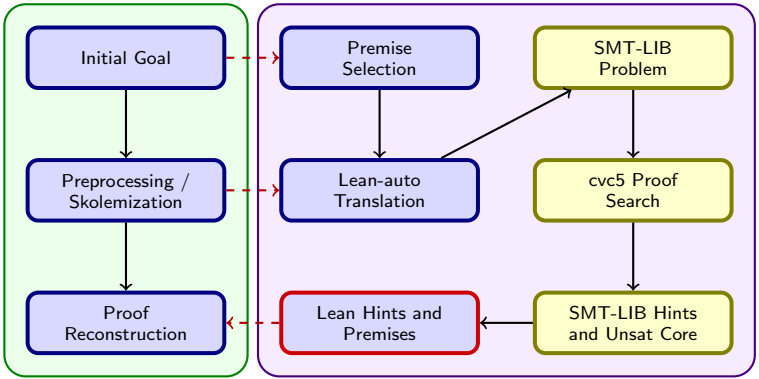

Hint Interpretation

Solver output (cvc5 hints)

```
preprocessing facts:    (=> (forall ((_i_0 Int)) ...)  ...)    ...      theory lemmas:    (let ((_let_1 ...))  ...)    ...
```

⇓

Initial Lean interpretation

```
have smtLemma0 :  ∀ (_i_0 :  ℤ), f sk0 ≤ f _i_0 → ¬f sk0 + -1 * f _i_0 ≥ Int.ofNat 1 := by grind
have smtLemma1 :  (∀ (_i_1 _i_2 :  ℤ), f _i_1 = f _i_2 → _i_1 = _i_2) → ∀ (_i_1 _i_2 :  ℤ), ¬f _i_1 = f _i_2 ∨ _i_1 = _i_2 := by grind
have smtLemma2 :  ∀ (bv0 :  ℤ), have _let_1 := sk1 ... _let_1 + -1 * f _let_1 ...
∀ (BOUND_VARIABLE_4962 :  ℤ), f BOUND_VARIABLE_4962 +.ofNat 1 ≤ f (sk1 BOUND_VARIABLE_4962) ≥ Int.ofNat 0 := by grind
have smtLemma3 :  ∀ (BOUND_VARIABLE_4953 :  ℤ), ¬BOUND_VARIABLE_4953 = sk1 BOUND_VARIABLE_4953 := by grind
have smtLemma4 :  have _let_1 := f sk0; have _let_2 := f (sk1 sk0);
(¬_let_1 + -1 * _let_2 ≥ Int.ofNat 0 ∨ _let_1 = _let_2) ∨ _let_1 + -1 * _let_2 ≥ Int.ofNat 1 := by grind
have smtLemma5 :  have _let_1 := f sk0; have _let_2 := f (sk1 sk0); have _let_3 := _let_1 + -1 * _let_2;
_let_3 < Int.ofNat 1 → (¬_let_3 ≥ Int.ofNat 0 ∨ _let_1 = _let_2) ∨ _let_3 ≥ Int.ofNat 1 := by grind
have smtLemma6 :  (sk1 sk0 = sk0) = (sk0 = sk1 sk0) := by grind
have smtLemma7 :  (f (sk1 sk0) = f sk0) = (f sk0 = f (sk1 sk0)) := by grind
```

Hint Interpretation

Solver output (cvc5 hints)

```
preprocessing facts:    (=> (forall ((_i_0 Int)) ...)  ...)    ...
theory lemmas:    (let ((_let_1 (_f _sk0_...)))  ...)    ...
```

⇓

Initial Lean interpretation

```
have smtLemma0 :  ∀ (_i_0 :  ℤ), f sk0 ∈ f _i_0 → ¬f sk0 → Int.ofNat ... := by grind
have smtLemma1 :  (∀ (_i_1 _i_2 :  ℤ), f _i_1 = f _i_2 → _i_1 = _i_2) → ... := by grind
have smtLemma2 :  ∀ (bv0 :  ℤ), have _let_1 := sk1 ... ∀ (BV_4962 :  ℤ), f BV_4962 +.ofNat 1 ... := by grind  ...
...
```

⇓

Refined Lean interpretation

```
have smtLemma0 :  ∀ (_i_0 :  ℤ), ¬ f sk0 + -1 * f _i_0 ≥ 1 := by grind
have smtLemma1 :  (∀ (_i_1 _i_2 :  ℤ), f _i_1 = f _i_2 → _i_1 = _i_2) → ... := by grind
have smtLemma2 :  ∀ (bv0 :  ℤ), f bv0 + -1 * f (sk1 bv0) ≥ 0 := by grind
have smtLemma3 :  ∀ (bv0 :  ℤ), ¬bv0 = sk1 bv0 := by grind
have smtLemma4 :  (¬f sk0 + -1 * f (sk1 sk0) ≥ 0 ∨ f sk0 = f (sk1 sk0)) ∨ f sk0 + -1 * f (sk1 sk0) ≥ 1 := by grind
have smtLemma5 :  f sk0 + -1 * f (sk1 sk0) < 1 → ... := by grind
have smtLemma6 :  (sk1 sk0 = sk0) = (sk0 = sk1 sk0) := by grind
have smtLemma7 :  (f (sk1 sk0) = f sk0) = (f sk0 = f (sk1 sk0)) := by grind
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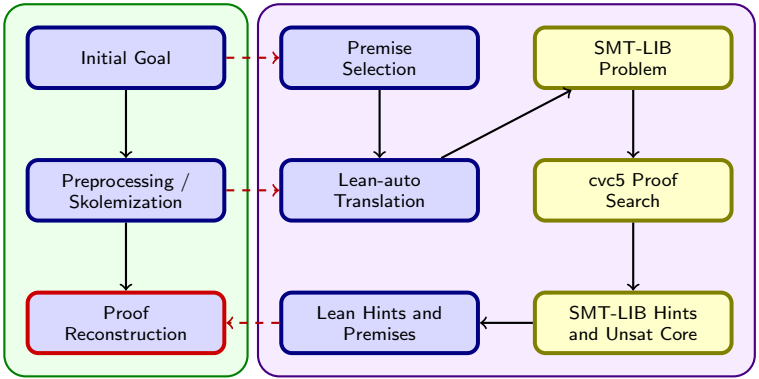

Proof Reconstruction

```
example (f : ℤ → ℤ) (h1: ∀ x y, f x = f y → x = y)
(h2 : ∃ x, ∀ y, f x ≤ f y) :
∃ x, ∀ (y : ℤ), x ≠ y → f x < f y := by
@[classical] byContradiction
intro negGoal
skolemizeAll
```

```
[querySMT.debug] getDuperCoreSMTLemmas ::
Calling runDuperPortfolioMode with formulas:
[∀ (x : ℤ), ¬(x ≠ sk1 x → f x < f (sk1 x)),
∀ (y : ℤ), f sk0 ≤ f y,
∀ (x y : ℤ), f x = f y → x = y,
∀ (_i_0 : ℤ), ¬f sk0 + -1 * f _i_0 ≥ 1,
∀ (_i_1 _i_2 : ℤ), ¬f _i_1 = f _i_2 ∨ _i_1 = _i_2,
∀ (bv0 : ℤ), f bv0 + -1 * f (sk1 bv0) ≥ 0,
∀ (bv0 : ℤ), ¬bv0 = sk1 bv0,
(¬f sk0 + -1 * f (sk1 sk0) ≥ 0 ∨ f sk0 = f (sk1
sk0))
∨ f sk0 + -1 * f (sk1 sk0) ≥ 1,
f sk0 + -1 * f (sk1 sk0) < 1 → ...,
(sk1 sk0 = sk0) = (sk0 = sk1 sk0),
(f (sk1 sk0) = f sk0) = (f sk0 = f (sk1 sk0))]
```

Proof Reconstruction

```
example (f : ℤ → ℤ) (h1: ∀ x y, f x = f y → x = y)
(h2 : ∃ x, ∀ y, f x ≤ f y) :
∃ x, ∀ (y : ℤ), x ≠ y → f x < f y := by
@[classical] byContradiction
intro negGoal
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```

```
[querySMT.debug] getDuperCoreSMTLemmas ::
Calling runDuperPortfolioMode
with formulas:
[∀ (x : ℤ), ¬(x ≠ sk1 x → ...),
∀ (y : ℤ), f sk0 ≤ f y,
∀ (x y : ℤ), f x = f y → x = y,
...]
```

→

```
[querySMT.debug] ∀ (_i_0 : ℤ), ...
appears in the proof (index: 0)
[querySMT.debug] ∀ (_i_1 _i_2 : ℤ), ...
does not appear in the proof (index: 1)
[querySMT.debug] ∀ (bv0 : ℤ), ... appears (2)
[querySMT.debug] ∀ (bv0 : ℤ), ... appears (3)
[querySMT.debug] (¬f sk0 ...) appears (4)
[querySMT.debug] f sk0 ... does not appear (5)
[querySMT.debug] (sk1 sk0 = sk0) ...not appear (6)
[querySMT.debug] (f (sk1 sk0) = f sk0) ... not (7)
```

Proof Reconstruction

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have smtLemma3 : ∀ (bv0 : ℤ), ¬bv0 = sk1 bv0 := by grind
have smtLemma4 : (¬f sk0 + -1 * f (sk1 sk0) ≥ 0 ∨ f sk0 = f (sk1 sk0))
∨ f sk0 + -1 * f (sk1 sk0) ≥ 1 := by grind
duper [h1, smtLemma0, smtLemma2, smtLemma3, smtLemma4] []
```

Conclusion

- New hint-based approach to leveraging SMT solvers for ITP automation
  - Translates Lean goals to SMT-LIB, extracts preprocessing and theory lemmas from cvc5, produces self-contained proof scripts after minimization
- Evaluated on Int, Nat, and List benchmarks from Mathlib
  - QuerySMT compares favorably to existing SMT-related Lean automation
  - Hints produce a clear improvement in the underlying proof automation
  - Only ∼5% of hints fail to be certified by `grind`
- Future work
  - Support more SMT theories beyond integers and algebraic datatypes
  - Collect quantifier instances from cvc5 proofs to speed up Duper
  - Better instrumentation for AC operator normalization
  - Port to other proof assistants / SMT solvers

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Experimental Results

- **lean-auto + cvc5**: translates to SMT-LIB and trusts cvc5 proofs (theoretical upper bound)
- **querysmt**: full pipeline with hints from cvc5
- **querysmt$^-$**: like querysmt but without cvc5 hints (only unsat core minimization)
- **lean-smt**: proof replay approach for cvc5 in Lean
- **grind**: built-in Lean tactic inspired by SMT solvers (no external solver)

	Int Theorems	**Nat Theorems**	**List Theorems**
Total	2058	3270	4576
lean-auto + cvc5	1137	1486	891
querysmt	840	892	749
querysmt$^-$	472	627	708
lean-smt	333	35	445
grind	541	812	–