SMT solving for fun and profit

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UFMG

BIRS Workshop 26w5626
Theory and Practice of SAT and Combinatorial Solving

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Acknowledgments

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Disclaimer: The literature on SMT and its applications is vast. The bibliographic references provided here are just a small and highly incomplete sample. Apologies to all authors whose work is not cited.

Agenda

Introduction

Automated Reasoning for Formal Methods

Two successful examples:

SAT: propositional formalization, Boolean reasoning

- \+ high degree of efficiency
- – expressive (all NP-complete problems) but involved encodings

SMT: first-order formalization, Boolean + domain-specific reasoning

- \+ improves expressivity and scalability
- – some (but acceptable) loss of efficiency

The Basic SMT Problem

Determining the satisfiability of a logical formula wrt some combination T of background theories

Example

$$n > 3 * m + 1 \;\wedge\; (\; f\,(n) \leq head\,(l_1) \;\vee\; l_2 = f\,(n) :: l_1 \;)$$

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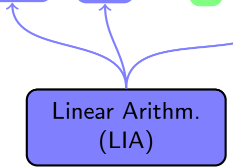

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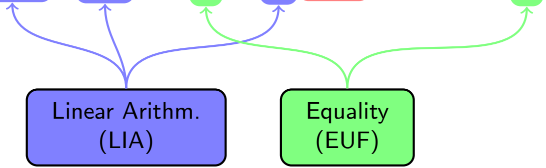

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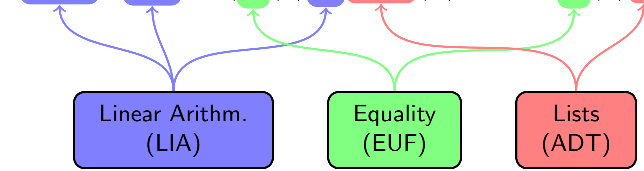

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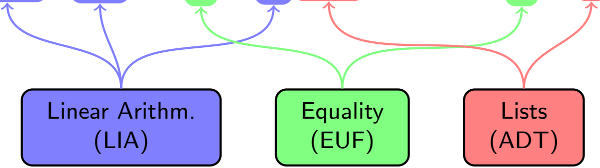

SMT formulas are formulas in
many-sorted FOL with built-in symbols

SMT solvers

Are highly efficient tools for the SMT problem based on specialized logic engines

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Are changing the way people solve problems in Computer Science and beyond:

- instead of building a special-purpose tool
- translate problem into a logical formula
- use an SMT solver as backend reasoner

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- translate problem into a logical formula
- use an SMT solver as backend reasoner

Not only easier, often better

Some Applications of SMT

Model Checking

- (in)finite-state systems
- hybrid systems
- abstraction refinement
- state invariant generation
- interpolation

Type Checking

- dependent types
- semantic subtyping
- type error localization

Program Analysis

- symbolic execution
- program verification
- verification in separation logic
- (non-)termination
- loop invariant generation
- procedure summaries
- race analysis
- concurrency errors detection

Software Synthesis

- syntax-guided function synthesis
- automated program repair
- synthesis of reactive systems
- synthesis of self-stabilizing systems
- network schedule synthesis

More Applications of SMT

Security
- automated exploit generation
- protocol debugging
- protocol verification
- analysis of access control policies
- run-time monitoring

Compilers
- compilation validation
- optimization of arithmetic computations

Planning
- motion planning
- nonlinear PDDL planning

Software Engineering
- system model consistency
- design analysis
- test case generation
- verification of ATL transformations
- semantic search for code reuse
- interactive (software) requirements prioritization
- generating instances of meta-models
- behavioral conformance of web services

Machine Learning
- verification of deep NNs

Business
- verification of business rules
- spreadsheet debugging

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Heavily used at AWS

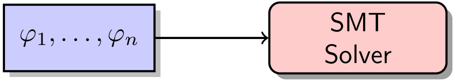
Billions SMT queries a day via Zelkova[a]

[a] Backes et al. 2018; Rungta 2022

SMT solver functionality

SMT Solver Basic Functionality

Background theory T



Uninterpreted Funs	$x = y \Rightarrow f(x) = f(y)$
Integer/Real Arithmetic	$2x + y = 0 \wedge 2x - y = 4 \rightarrow x = 1$
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Bit-vectors	$4 \cdot (x \gg 2) = x \mathbin{\&} {\sim}3$
Strings and RegExs	$x = y \cdot z \wedge z \in ab^* \Rightarrow \lvert x \rvert > \lvert y \rvert$
Arrays	$i = j \Rightarrow \mathsf{store}(a, i, x)[j] = x$

SMT Solver Basic Functionality

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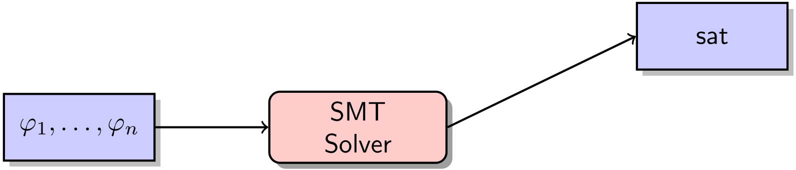

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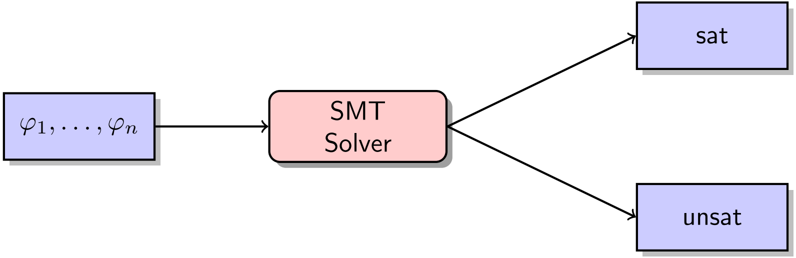

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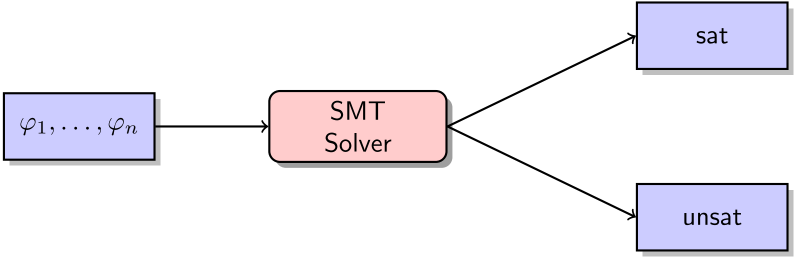

sat/unsat: there is a/no model M of T such that

$$M \models \varphi_1 \wedge \cdots \wedge \varphi_n$$

SMT Solver Basic Functionality

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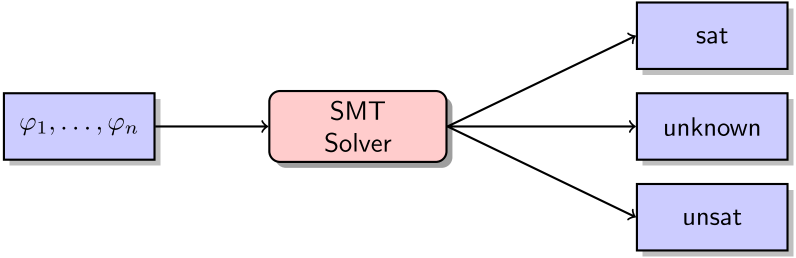


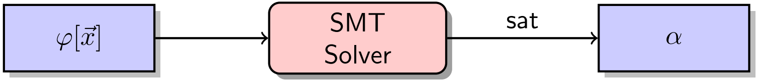
sat/unsat: there is a/no model M of T such that

$$M \models \varphi_1 \wedge \cdots \wedge \varphi_n$$

unknown: inconclusive — because of resource limits or incompleteness

SMT Solver Output: Satisfying Assignments

Background theory T



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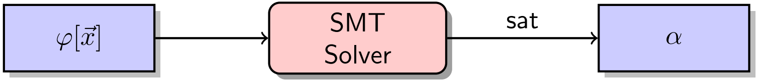

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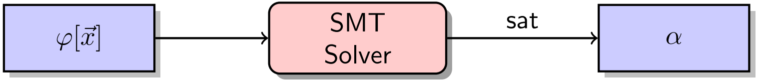

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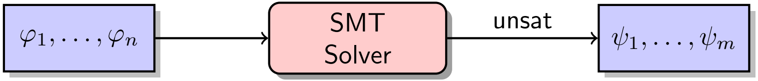
Note.

$\vec{x}$ may consist of first- and second-order variables
(aka, uninterpreted constants and function symbols)

SMT Solver Output: Unsat Cores

Background theory T



$\psi_1, \ldots, \psi_m$ is a unsat core of $\{\varphi_1, \ldots, \varphi_n\}$:

SMT Solver Output: Unsat Cores

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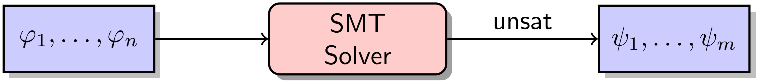
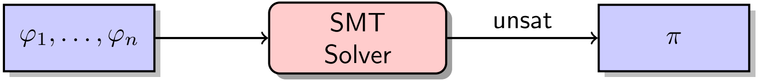


$\psi_1, \ldots, \psi_m$ is a unsat core of $\{\varphi_1, \ldots, \varphi_n\}$:

1. $\{\psi_1, \ldots, \psi_m\} \subseteq \{\varphi_1, \ldots, \varphi_n\}$
2. $\{\psi_1, \ldots, \psi_m\}$ is unsat in T
3. $\{\psi_1, \ldots, \psi_m\}$ is minimal (or smallish)

SMT Solver Output: Proofs

Background theory T



π is a checkable proof object for $\{\varphi_1, \ldots, \varphi_n\}$:

SMT Solver Output: Proofs

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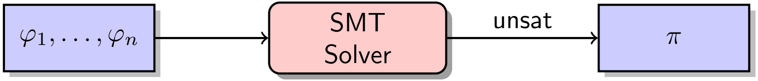

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1. π is a proof term in some formal proof system
2. π expresses a refutation of $\{\varphi_1, \ldots, \varphi_n\}$
3. π can be efficiently checked by an external proof checker

SMT Solver Output: Proofs

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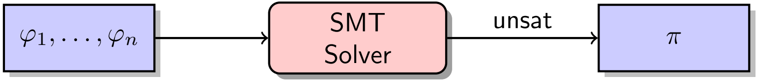

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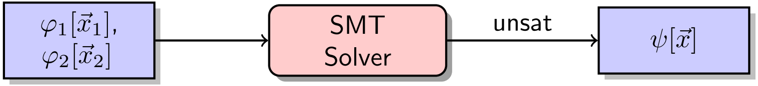
3. π can be efficiently checked by an external proof checker
   - The "efficiently" there is actually a highly debatable point...

Extended Functionality: Interpolation

Background theory T



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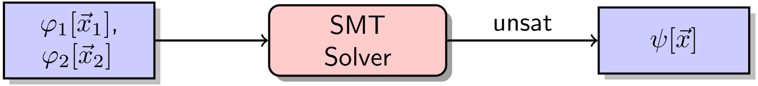
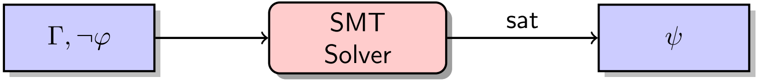


ψ is a logical interpolant of φ_1 and φ_2:

1. $\varphi_1 \models_T \psi$ and $\psi \models_T \neg\varphi_2$
2. $\vec{x} = \vec{x}_1 \cap \vec{x}_2$

Extended Functionality: Abduction

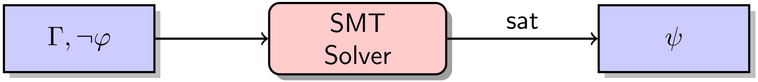
Background theory T



ψ is an abduction hypothesis for φ wrt Γ:

Extended Functionality: Abduction

Background theory T

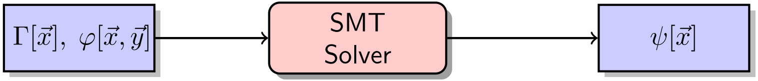


ψ is an abduction hypothesis for φ wrt Γ:

1. Γ, ψ is satisfiable in T
2. $\Gamma, \psi \models_T \varphi$
3. ψ is maximal, e.g., with respect to $\models_T$
   (if ψ' satisfies 1 and 2 and $\psi \models_T \psi'$ then $\psi' \models_T \psi$)

Extended Functionality: Quantifier Elimination

Background theory T



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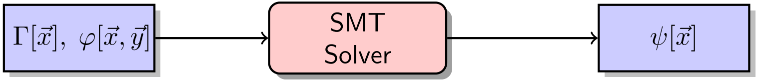

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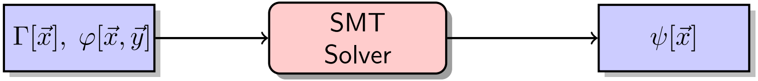

ψ is a projection of φ over $\vec{y}$ with respect to Γ:

1. $\Gamma \models_T \psi \Leftrightarrow \exists \vec{y}\, \varphi$

Extended Functionality: Optimization

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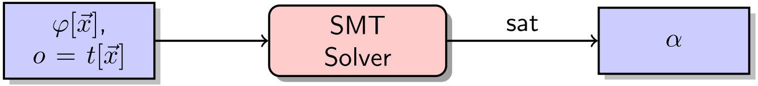

α is a an optimal assignment for φ:

Extended Functionality: Optimization

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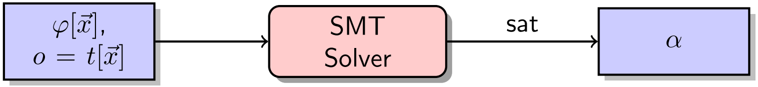

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2. $M \models \varphi[\vec{x} \mapsto \vec{v}]$ for some model M of T
3. α minimizes/maximizes objective o

Background theories

Background Theories

Uninterpreted Funs	$x = y \Rightarrow f(x) = f(y)$
Integer/Real Arithmetic	$2x + y = 0 \wedge 2x - y = 4 \rightarrow x = 1$
Floating Point Arithmetic	$x + 1 \neq NaN \wedge x < \infty \Rightarrow x + 1 > x$
Bit-vectors	$4 \cdot (x \gg 2) = x \,\&\, {\sim}3$
Strings and RegExs	$x = y \cdot z \wedge z \in ab^* \Rightarrow \lvert x \rvert > \lvert y \rvert$
Arrays	$i = j \Rightarrow \mathsf{store}(a, i, x)[j] = x$
Algebraic Data Types	$x \neq \mathit{Leaf} \Rightarrow \exists\, l, r : \mathit{Tree}(\alpha).\ \exists\, a : \alpha.\ x = \mathit{Node}(l, a, r)$
Finite Sets	$e_1 \in x \wedge e_2 \in x \setminus e_1 \Rightarrow \exists y, z : \mathit{Set}(\alpha).\ \lvert y \rvert = \lvert z \rvert \wedge x = y \cup z \wedge y \neq \emptyset$
Finite Relations	$(x, y) \in r \wedge (y, z) \in r \Rightarrow (x, z) \in r \bowtie s$

Equality and Uninterpreted Functions (EUF)(Nelson and Oppen 1980; Nieuwenhuis and Oliveras 2007)

Simplest first-order theory with equality, applications of uninterpreted functions, and variables of uninterpreted sorts

For all sorts σ, σ' and function symbols $f : \sigma \to \sigma'$

Reflexivity: $\forall x : \sigma.\ x = x$
Symmetry: $\forall x : \sigma.\ x = y \Rightarrow y = x$
Transitivity: $\forall x, y : \sigma.\ x = y \land y = z \Rightarrow x = z$
Congruence: $\forall \vec{x}, \vec{y} : \vec{\sigma}.\ \vec{x} = \vec{y} \Rightarrow f(\vec{x}) = f(\vec{y})$

Congruence closure decision procedure can efficiently handle conjunctions of equality literals.

Example

$$f(f(f(a))) = b \qquad g(f(a), b) = a \qquad f(a) = a$$

Arrays

(Bofill et al. 2008; McCarthy 1993; Moura and Bjørner 2009; Stump et al. 2001)

Operates over sorts $\mathsf{Array}(\sigma_i, \sigma_e)$, σ_i, σ_e and function symbols

$$\_[\_] : \mathsf{Array}(\sigma_i, \sigma_e) \times \sigma_i \to \sigma_e$$
$$\mathsf{store} : \mathsf{Array}(\sigma_i, \sigma_e) \times \sigma_i \times \sigma \to \mathsf{Array}(\sigma_i, \sigma_e)$$

For any index sort σ_i and element sort σ_e

Read-Over-Write-1: $\forall a, i, e.\ \mathsf{store}(a, i, e)[i] = e$

Read-Over-Write-2: $\forall a, i, j, e.\ i \neq j \Rightarrow \mathsf{store}(a, i, e)[j] = a[j]$

Extensionality: $\forall a, b, i.\ a \neq b \Rightarrow \exists i.\ a[i] \neq b[i]$

Efficient decision procedure based on congruence closure to handle equality reasoning and strong filters for restricting the application of inferences capturing the above axioms.

Example

$$\mathsf{store}(\mathsf{store}(a, i, a[j]), j, a[i]) = \mathsf{store}(\mathsf{store}(a, j, a[i]), i, a[j])$$

Arithmetic

Restricted fragments, over the reals or the integers, support efficient methods:

- Bounds: $x \bowtie k$ with $\bowtie \in \{<, >, \leq, \geq, =\}$ (Bozzano et al. 2005)
- Difference constraints: $x - y \bowtie k$, with $\bowtie \in \{<, >, \leq, \geq, =\}$ (Cotton and Maler 2006; Nieuwenhuis and Oliveras 2005; Wang et al. 2005)
- UTVPI: $\pm x \pm y \bowtie k$, with $\bowtie \in \{<, >, \leq, \geq, =\}$ (Lahiri and Musuvathi 2005)
- Linear arithmetic, e.g: $2x - 3y + 4z \leq 5$ (Bjørner and Nachmanson 2024; Dutertre and Moura 2006)
- Non-linear arithmetic, e.g: $2xy + 4xz^2 - 5y \leq 10$ (Ábrahám et al. 2021; Borralleras et al. 2009; Jovanović and Moura 2012; Zankl and Middeldorp 2010)

Example

Are there real solutions for $x^2y + yz + 2xyz + 4xy + 8xz + 16 = 0$?

Machine Arithmetic — Bit-vectors (Brummayer and Biere 2009; Niemetz and Preiner 2023)

Combines arithmetic operations, bit-wise operations, shift, extraction, concatenation.

Most effective decision procedures rely primarily on bit-blasting, i.e., converting the bit-vector problem to an equisatisfiable Boolean representation and leveraging state-of-the-art SAT solvers.

Example

Consider the following implementations of the absolute value opeartor for 32-bit integers:

0. $abs_0(x) := x < 0\ ?\ -x : x$
1. $abs_1(x) := (x \oplus (x \gg_a 31)) - (x \gg_a 31)$
2. $abs_2(x) := (x + (x \gg_a 31)) \oplus (x \gg_a 31)$
3. $abs_3(x) := x - ((x \ll 1)\ \&\ (x \gg_a 31))$

How do we prove that all four are equivalent to one another?

Machine Arithmetic – Floating-Points

FP in SMT

- Follows IEEE 754-2019
- FP number = triple of bit-vectors
- Wide range of operators
  - take a rounding mode as input
- E.g., addition, multiplication, fused-multiplication-addition
- As with bit-vectors, most effective procedures rely on bit-blasting.

Example

Is addition associative in floating-point arithmetic, i.e., is $a + (b + c) \neq (a + b) + c$ valid?

(Co-)Algebraic Data Types

(Barrett et al. 2007; Reynolds and Blanchette 2017)

Family of user-definable theories

Example

$$\text{Tree} \quad := \quad \text{nil} \mid \text{node}(\text{data} : \text{Int}, \text{left} : \text{Tree}, \text{right} : \text{Tree})$$

Distinctiveness: $\forall h, t.\ \text{nil} \neq h :: t$

Exhaustiveness: $\forall l.\ l = \text{nil} \vee \exists h, t.\ h :: t$

Injectivity: $\forall h_1, h_2, t_1, t_2.$
$$h_1 :: t_1 = h_2 :: t_2 \Rightarrow h_1 = h_2 \wedge t_1 = t_2$$

Selectors: $\forall h, t.\ \text{head}(h :: t) = h \wedge \text{tail}(h :: t) = t$

(**Non-circularity:** $\forall l, x_1, \ldots, x_n.\ l \neq x_1 :: \cdots :: x_n :: l$)

Strings and regular expressions (Abdulla et al. 2015; Kiezun et al. 2009; Liang et al. 2014)

SMT Strings

- Represent common programming languages Unicode strings
- Supports a wide range of operators
  - concatenation, length, substring, etc
- Regular expressions crucial for some applications, such as analysis of access control policies

Example

Can we have a string with at most three characters that also contains the string "BIRS"?

Other Interesting Theories

- Finite sets with cardinality (Bansal et al. 2016)
- Finite relations (Meng et al. 2017)
- Transcendental Functions (Cimatti et al. 2017b; Gao et al. 2013)
- Ordinary differential equations (Gao et al. 2013)
- Finite Fields (Hader et al. 2023; Ozdemir et al. 2023)
- ...

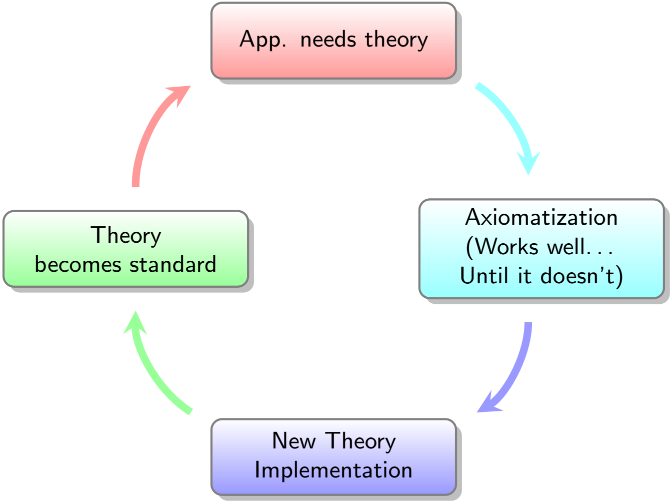
Some SMT solvers also allow you to axiomatize your own theory

- The effective procedures discussed so far generally assume quantifier-free logical fragments
- However new applications may not fit directly into existing theories, which necessitates reasoning about user-defined axioms
- Some solvers (notably, cvc5, veriT, and Z3) support them, but this support has caveats
  - Undecidable in general
  - Explosive heuristics
  - Users want it to work as well as on quantifier-free problems

Example

What if we did not have a theory of arrays but wanted to reason about them?

The SMT Cycle



Application example: Software Verification

Software Verification

Example

```
void swap(int* a, int* b) {
  *a = *a + *b;
  *b = *a - *b;
  *a = *a - *b;
}
```

Check if the swap is correct:

- Heap: $Array(BV_{32}) \mapsto BV_{32}$
- Update heap line by line
- Check that
  a* = old(b*) and b* = old(a*)

Software Verification

Example

```
void swap(int* a, int* b) {
  *a = *a + *b;
  *b = *a - *b;
  *a = *a - *b;
}
```

Check if the swap is correct:

- Heap: $Array(BV_{32}) \mapsto BV_{32}$
- Update heap line by line
- Check that
  a* = old(b*) and b* = old(a*)

$$h_1 = \mathsf{store}(h_0, a, h_0[a] +_{32} h_0[b])$$
$$h_2 = \mathsf{store}(h_1, b, h_1[a] -_{32} h_1[b])$$
$$h_3 = \mathsf{store}(h_2, a, h_2[a] -_{32} h_2[b])$$
$$\neg(h_3[a] = h_0[b] \land h_3[b] = h_0[a])$$

Example

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void swap(int* a, int* b) {
  *a = *a + *b;
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Check if the swap is correct:

- Heap: $Array(BV_{32}) \mapsto BV_{32}$
- Update heap line by line
- Check that
  a* = old(b*) and b* = old(a*)
- Incorrect: aliasing

SMT solver solution

$a \mapsto 0, \quad b \mapsto 0$
$h_0[0] \mapsto 1, \quad h_1[0] \mapsto 2$
$h_2[0] \mapsto 0, \quad h_3[0] \mapsto 0$

${}_{32}\ h_0[b])$

${}_{32}\ h_1[b])$

$h_3 = \text{store}(h_2, a, h_2[a] -_{32} h_2[b])$

$\neg(h_3[a] = h_0[b] \wedge h_3[b] = h_0[a])$

Contract-based Software Verification

Example (Binary Search)

```
//@assume 0 <= n <= |a| &&
//          foreach i in [0..n−2]. a[i] <= a[i+1]
//@ensure (0 <= res ==> a[res] = k) &&
//          (res < 0 ==> foreach i in [0..n−1]. a[i] != k)
int BinarySearch(int[] a, int n, int k) {
  int l = 0;  int h = n;
  while (l < h) { // Find middle value
    //@invariant 0 <= low < high <= len <= |a| &&
    //              foreach i in [0..low−1]. a[i]<k &&
    //              foreach i in [high..len−1]. a[i] > k
    int m = l + (h − l) / 2;  int v = a[m];
    if (k < v) { l = m + 1; } else if (v < k) { h = m; }
    else { return m; }
  }
  return −1;
}
```

Example adapted from Moura and Bjørner 2010

Example (Binary Search)

Main approach

1. Compile source and annotations to a series of pre-conditions, commands over the state, and post-conditions.
2. Generate verification conditions on SMT

```
//@as
//
//@e
//
int B
  int l = 0;   int h = n;
  while (l < h) { // Find middle value
    //@invariant 0 <= low < high <= len <= |a| &&
    //             foreach i in [0..low-1]. a[i]<k &&
    //             foreach i in [high..len-1]. a[i] > k
    int m = l + (h - l) / 2;  int v = a[m];
    if (k < v) { l = m + 1; } else if (v < k) { h = m; }
    else { return m; }
  }
  return -1;
}
```

Example adapted from Moura and Bjørner 2010

Contract-based Software Verification

$$pre = 0 \leq n \leq |a| \wedge \forall i : \mathsf{Int}\ \ 0 \leq i \wedge i \leq n - 2 \Rightarrow a[i] \leq a[i+1]$$

$$\begin{aligned} post = {} & (0 \leq res \Rightarrow a[res] = k) \wedge \\ & (res < 0 \Rightarrow \forall i : \mathsf{Int}\ 0 \leq i \wedge i \leq n - 1 \Rightarrow a[i] \neq k) \end{aligned}$$

$$\begin{aligned} inv = {} & 0 \leq l \wedge l \leq h \wedge h \leq n \wedge n \leq |a| \wedge \\ & \forall i : \mathsf{Int}\ \ 0 \leq i \wedge i \leq l - 1 \Rightarrow a[i] < k \wedge \\ & \forall i : \mathsf{Int}\ \ h \leq i \wedge i \leq n - 1 \Rightarrow a[i] > k \end{aligned}$$

Contract-based Software Verification

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$$\begin{aligned} & pre \land \neg \mathbf{let}\ l = 0,\ h = n\ \mathbf{in}\ inv \land \forall l, h : \mathsf{Int}\ inv \Rightarrow \\ & \quad (\neg(l < h) \Rightarrow post\{res \mapsto -1\}) \land \\ & \quad (l < h \Rightarrow \mathbf{let}\ m = l + (h-l)/2,\ v = a[m]\ \mathbf{in} \\ & \qquad (k < v \Rightarrow inv\{l \mapsto m+1\}) \land \\ & \qquad (\neg(k < v) \land v < k \Rightarrow inv\{n \mapsto m\}) \land \\ & \qquad (\neg(k < v) \land \neg(v < k) \Rightarrow post\{res \mapsto m\})) \end{aligned}$$

$$pre = 0 \leq n \leq |a| \land \forall i : \mathsf{Int}\ \ 0 \leq i \land i \leq n - 2 \Rightarrow a[i] \leq a[i+1]$$

$$post = (0 \leq res \Rightarrow a[res] = k) \land$$
$$(res < 0 \Rightarrow \forall i : \mathsf{Int}\ 0 \leq i \land i \leq n - 1 \Rightarrow a[i] \neq k)$$

$$inv = 0 \leq l \land l \leq h \land h \leq n \land n \leq |a| \land$$
$$\forall i : \mathsf{Int}\ \ 0 \leq i \land i \leq l - 1 \Rightarrow a[i] < k \land$$
$$\forall i : \mathsf{Int}\ \ h \leq i \land i$$

$$pre \land \neg \mathbf{let}\ l = 0,\ h = n$$
$$(\neg(l < h) \Rightarrow post\{res \mapsto -1\}) \land$$
$$(l < h \Rightarrow \mathbf{let}\ m = l + (h - l)/2,\ v = a[m]\ \mathbf{in}$$
$$(k < v \Rightarrow inv\{l \mapsto m + 1\}) \land$$
$$(\neg(k < v) \land v < k \Rightarrow inv\{n \mapsto m\}) \land$$
$$(\neg(k < v) \land \neg(v < k) \Rightarrow post\{res \mapsto m\}))$$

SMT solver answer

Unsatisfiable

What's next? What's hot?

Stability

- Seemingly irrelevant changes (e.g. variable renaming, assertion order) can have unpredictable performance impact.
- For the longest time SMT developers could get away with "these are NP-complete or undecidable problems, we must use heuristics, so this is to be expected."
- Users have been pushing back more and more so we had to accept we need to deal with this.
- Initial explorations:
  - Normalization of input (Amrollahi et al. 2025)
  - Identifying missing instances across mutated input (Zhou et al. 2025)
  - Better engineering (Cebeci et al. 2025)

Improving hard theories

- Bitvectors (BV) and floating-point arithmetic (FP)
  - Leveraging numerical methods in FP (Zhang et al. 2026)
  - Algebraic methods for BV (Kaufmann and Biere 2021; Rath et al. 2024)
  - CEGAR-based approaches for mitigating bit-blasting bottlenecks (Niemetz et al. 2024)
- Strings
  - Automata-based approaches in Z3Noodler (Chocholatý et al. 2025) and OSTRICH (Hague et al. 2025)
  - Symbolic derivatives in Z3 (Varatalu et al. 2025)
- Non-linear arithmetic
  - Cylindrical algebraic decomposition (CAD) based methods in SMT-RAT (Ábrahám et al. 2021; Promies et al. 2025)
  - MCSat-based methods in Yices (Lipparini et al. 2025)
  - CAD and incremental linearization methods in cvc5 (Cimatti et al. 2017a; Kremer et al. 2022)

Proofs

- Numerous applications (correctness, integration with other systems, interpretability)
  - Improving automation in Lean (Mohamed et al. 2025) and Isabelle (Lachnitt et al. 2025)
  - Huge investment from AWS aiming to automate compliance via cvc5 proofs (Barbosa et al. 2023)
  - cvc5 proofs may be going into the Linux kernel (Sun and Su 2025)

- Numerous challenges
  - Enormous effort to instrument solvers to produce *detailed* proofs (Barbosa et al. 2022)
  - No standard format or proof calculus, leading to lots of duplicated work (Hoenicke and Schindler 2022; Moura and Bjørner 2008; Schurr et al. 2021)
  - Proofs for complex theory solvers (e.g. CAD-based, automata-based) and rewriters (e.g. strings) are not as mature
  - Huge proofs that are costly to produce and costly to check
    - Lazy proof generation can help (Hitarth et al. 2024)
    - Shorter proofs via different proof calculi (Liew et al. 2020) or algorithms (Andreotti and Barbosa 2026)

Others

- Parallel solving
  - Initial attempts of lifting cube-and-conquer to SMT (Hyvärinen et al. 2021; Wilson et al. 2023)
  - Clause-sharing in portfolio solving (Barrett et al. 2024)
- Using a SAT solver with chronological backtracking
  - ...

Thanks!

SMT solving for fun and profit

Haniel Barbosa

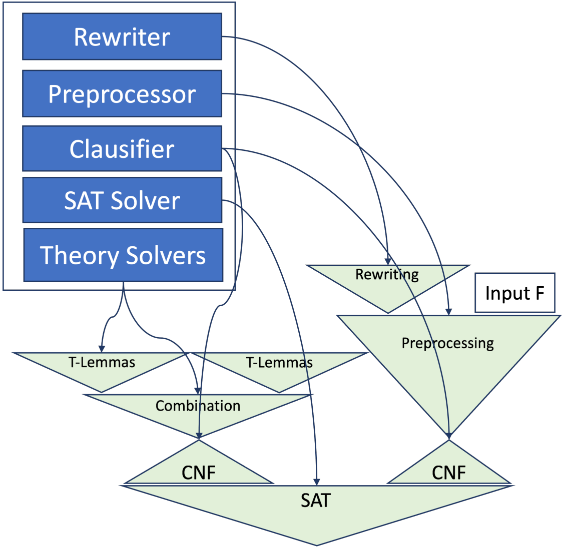
UFMG

BIRS Workshop 26w5626

Theory and Practice of SAT and Combinatorial Solving

Jan 13, 2026

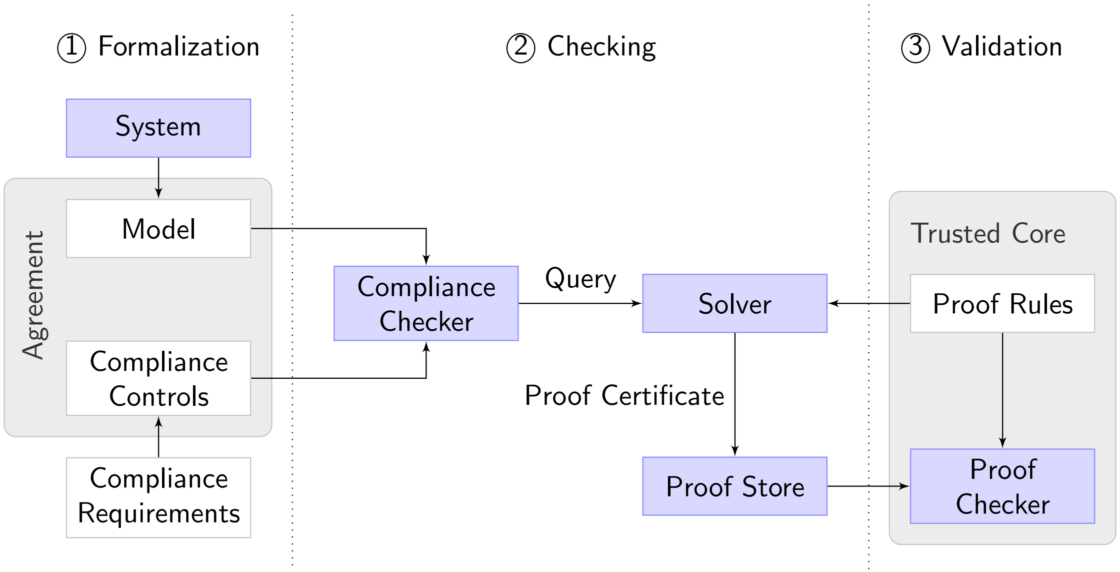
Resulting proofs



- ▷ Preprocessing
- ▷ Clausification
- ▷ Propositional reasoning
- ▷ Theory reasoning (UF, LIRA, Strings, . . .) and quantifier instantiation
- ▷ Theory combination
- ▷ Rewriting

Automated Compliance

(Barbosa et al. 2023)



Bounded Model Checking

To check the reachability of a class S of bad states for a system model M:

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   where
   - I encodes M's initial state condition and
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4. Encode S as a T-formula $B[\vec{x}]$
5. Find a k such that $I[\vec{x}_0] \wedge R[\vec{x}_0, \vec{x}_1] \wedge \cdots \wedge R[\vec{x}_{k-1}, \vec{x}_k] \wedge B[\vec{x}_k]$ is satisfiable in T

Bounded Model Checking

We can for example check if safety property P holds for 10 iterations.

- Unroll the loop 10 times or until property P is violated
- Check for each iteration if property P holds

C Code

```
int main () {
  bool turn;                  // input
  uint32_t a = 0, b = 0;      // states
  for (;;) {
   turn = read_bool ();
   assert (a != 3 || b != 3); // property P
   if (turn) a = a + 1;       // next(a)
   else      b = b + 1;       // next(b)
  }
}
```

Unroll

$a_0 = 0 \wedge b_0 = 0$

...check if P holds for a_0, b_0

$a_1 = next(a_0) \wedge b_1 = next(b_0)$

...check if P holds for a_1, b_1

$a_2 = next(a_1) \wedge b_2 = next(b_1)$

...check if P holds for a_2, b_2

$\ldots$

Symbolic Model Checking

To check the invariance of a state property S
for a system model M:

1. Choose a theory T decided by an SMT solver
   (e.g., quantifier-free linear arithmetic and EUF)

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   where
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4. Encode S as a T-formula $P[\vec{x}]$

5. Prove that $P[\vec{x}]$ holds in all reachable states of $(I[\vec{x}], R[\vec{x}, \vec{x}'])$

Symbolic Model Checking

Example: *Parametric Resettable Counter*

System

Vars

input pos int, n_0
input bool r
int c, n

Initialization

c $:=$ 1
n $:=$ n_0

Transitions

n' $:=$ n
c' $:=$ if (r' or c $=$ n)
then 1
else c $+$ 1

Property

$c \leq n + 1$

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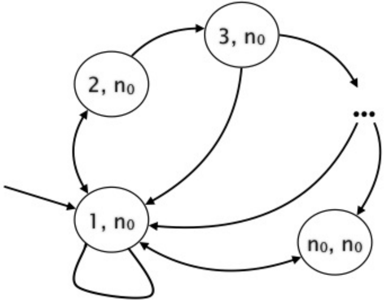

The transition relation contains infinitely many instances of the schema above, one for each $n_0 > 0$

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else c + 1

Property

$c \leq n + 1$

Encoding in T = LIA

$$\begin{aligned}
\vec{x} &:= (c, n, r, n_0) \\
I[\vec{x}] &:= c = 1 \\
&\wedge\ n = n_0 \\
R[\vec{x}, \vec{x}'] &:= n' = n \\
&\wedge\ (\neg r' \wedge c \neq n \vee c' = 1) \\
&\wedge\ (r' \vee c = n \vee c' = c + 1) \\
P[\vec{x}] &:= c \leq n + 1
\end{aligned}$$

Inductive Reasoning

$M = (I[\vec{x}], R[\vec{x}, \vec{x}'])$

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To prove $P[x]$ invariant for M it suffices
to show that it is inductive for M,
i.e.,

(1) $I[\vec{x}] \models_T P[\vec{x}]$ (base case)
and
(2) $P[\vec{x}] \land R[\vec{x}, \vec{x}'] \models_T P[\vec{x}']$ (inductive step)

Problem: Not all invariants are inductive

For the parametric resettable counter,
$P := c \leq n + 1$ is invariant but (2) is falsifiable
e.g., by $(c, n, r) = (4, 3, false)$ and $(c, n, r)' = (5, 3, false)$

$M = (I[\vec{x}$

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Strengthening Inductive Reasoning

(1) $I[\vec{x}] \models_T P[\vec{x}]$ (2) $P[\vec{x}] \wedge R[\vec{x}, \vec{x}'] \models_T P[\vec{x}']$

Various approaches:

Strengthening Inductive Reasoning

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Various approaches:

Strengthen P: find a property Q such that $Q[\vec{x}] \models_T P[\vec{x}]$ and prove Q inductive
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Strengthen R: find an auxiliary invariant $Q[\vec{x}]$ and use $Q[\vec{x}] \wedge R[\vec{x}, \vec{x}'] \wedge Q[\vec{x}']$ instead of $R[\vec{x}, \vec{x}']$
(ex:, Houdini, invariant sifting)

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Lengthen R: Consider increasingly longer R-paths $R[\vec{x}_0, \vec{x}_1] \wedge \cdots \wedge R[\vec{x}_{k-1}, \vec{x}_k] \wedge R[\vec{x}_k, \vec{x}_{k+1}]$
(ex:, k-induction)

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