Scalable Algorithms for Abduction via Enumerative Syntax-Guided Synthesis

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Overview

Abduction

Abduction

"What facts am I missing to reach a conclusion?"

The abduction problem

Given *axioms* Ax and a *goal* G, find a solution S such that:

- $\mathsf{Ax} \wedge \mathsf{S} \models \mathsf{G}$,
- $\mathsf{Ax} \wedge \mathsf{S}$ is satisfiable

Some applications of abduction

- ▷ Finding missing facts for discharging proof obligations [DDA12]
- ▷ Inferring library specifications [ZDD13]
- ▷ Synthesizing specifications for unknown subprocedures [ADG16]
- ▷ Loop invariant generation [DDLM13; EPS19]
- ▷ Compositional program verification [LDD+13]
- ▷ Synthesis of missing guards for memory safety [DDC14]
- ▷ ...

Despite numerous applications...

- Few standalone tools for abductive reasoning
  - GPID [EPS18]
  - EXPLAIN [DD13]

- Issues with
  - generality: restricted to specific logic fragments
  - flexibility: solutions within fixed criteria

Despite numerous applications...

- Few standalone tools for abductive reasoning
  - GPiD [EPS18]
  - Explain [DD13]

- Issues with
  - generality: restricted to specific logic fragments
  - flexibility: solutions within fixed criteria

Abduction via syntax-guided synthesis (SyGuS)

- Can be used with any background theory supported by SMT solvers
- Syntax restrictions can encode different criteria for solutions
- Standardized language: SyGuS-IF built on top of SMT-LIB

Redefining the abduction problem

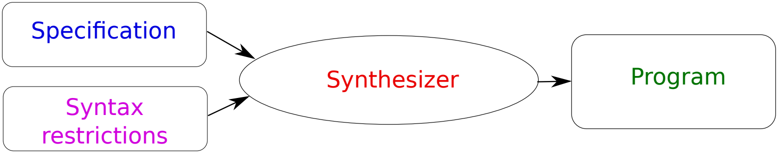
The (syntax-restricted) abduction problem for theory T

Given *axioms* Ax, *goal* G, theory T and grammar R, find a solution S such that:

- $\text{Ax} \wedge \text{S} \models_T \text{G}$,
- $\text{Ax} \wedge \text{S}$ is T-satisfiable
- S is generated by a context-free grammar R.

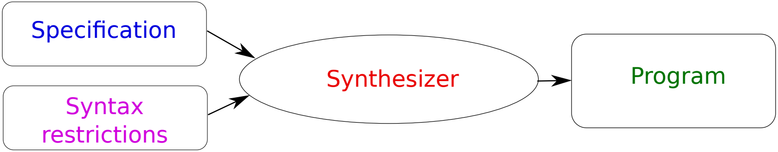
Abduction via Enumerative Syntax-Guided Synthesis (SyGuS)

Syntax-Guided Synthesis (SyGuS) [ABJ+13]



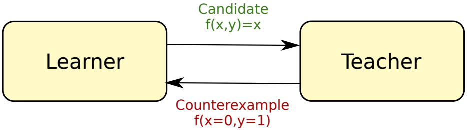
- ▷ Specification is given by (second-order) T-formula: $\exists f.\, \forall \bar{x}.\, \varphi[f, \bar{x}]$
- ▷ Syntactic restrictions given by context-free grammar R

Syntax-Guided Synthesis (SyGuS) [ABJ+13]



- ▷ Specification is given by (second-order) T-formula: $\exists f.\, \forall \bar{x}.\, \varphi[f, \bar{x}]$
- ▷ Syntactic restrictions given by context-free grammar R
- ▷ Commonly solved via enumerative CEGIS [STB+06; URD+13; RBN+19]

$I ::= 0 \mid 1 \mid x \mid y \mid I + I \mid I - I$



$\exists f.\, \forall \bar{x}.\, f(x, y) > x + 1$

Solving abduction with enumerative CEGIS

- We exploit the specification requiring that
  - $\mathsf{Ax}[\bar{x}] \land \mathsf{S}[\bar{x}] \land \neg\mathsf{G}[\bar{x}]$ be unsatisfiable

  to eagerly discard candidates

- Accumulate points (values $\bar{p}$ for $\bar{x}$) on which
  - Axioms are satisfied: $\textsc{eval}(\mathsf{Ax}[\bar{p}]) = \top$
  - Goal is falsified: $\textsc{eval}(\neg\mathsf{G}[\bar{p}]) = \top$

- Every candidate solution must be false on such points!
  - Otherwise $\textsc{eval}(\mathsf{Ax}[\bar{p}] \land \mathsf{S}[\bar{p}] \land \neg\mathsf{G}[\bar{p}]) = \top$

Solving abduction with enumerative CEGIS

Consider T as LIA, $\mathsf{Ax} = \{y \geq 0\}$, $\mathsf{G} = \{x + y + z \geq 0\}$,

$$R = \quad A \rightarrow 0 \mid 1 \mid x \mid y \mid z \mid A + A \qquad B \rightarrow A < A \mid A \geq A$$

and let P be the set of points satisfying Ax and falsifying G.

P	Candidate
$\{\}$	

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$x \geq 0 \wedge y \geq 0 \wedge x + y + z \not\geq 0$ is T-SAT, $p_1 = (0, 0, -1)$

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and let P be the set of points satisfying Ax and falsifying G.

P	Candidate
$\{p_1 = (0, 0, -1)\}$	$x < 0$

$\textsc{eval}(x < 0,\, p_1) = \bot$

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P	Candidate
$\{p_1 = (0, 0, -1)\}$	$x < 0$

$x < 0 \wedge y \geq 0 \wedge x + y + z \not\geq 0$ is T-SAT, $p_2 = (-1, 0, 0)$

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P	Candidate
$\{p_1 = (0,0,-1), p_2 = (-1,0,0)\}$	$y \geq 0$

$\textsc{eval}(y \geq 0,\, p_1) = \top, \textsc{eval}(y \geq 0,\, p_2) = \top$

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and let P be the set of points satisfying Ax and falsifying G.

P	Candidate
$\{p_1 = (0, 0, -1), p_2 = (-1, 0, 0)\}$	$y < 0$

$\textsc{eval}(y < 0,\ p_1) = \bot, \textsc{eval}(y < 0,\ p_2) = \bot$

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P	Candidate
$\{p_1 = (0, 0, -1), p_2 = (-1, 0, 0)\}$	$y < 0$

$y < 0 \land y \geq 0$ is not T-SAT

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$\{p_1 = (0, 0, -1), p_2 = (-1, 0, 0)\}$	$z \geq 0$

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$\{p_1 = (0, 0, -1), p_2 = (-1, 0, 0)\}$	$z \geq 0$

$\textsc{eval}(z \geq 0,\, p_1) = \bot, \textsc{eval}(z \geq 0,\, p_2) = \top$

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and let P be the set of points satisfying Ax and falsifying G.

P	Candidate
$\{p_1 = (0, 0, -1), p_2 = (-1, 0, 0)\}$	$z < 0$

$\text{EVAL}(z < 0,\, p_1) = \top, \text{EVAL}(z < 0,\, p_2) = \bot$

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and let P be the set of points satisfying Ax and falsifying G.

P	Candidate
$\{p_1 = (0, 0, -1), p_2 = (-1, 0, 0)\}$	$x + y \geq 0$

$\textsc{eval}(x + y \geq 0,\, p_1) = \top, \textsc{eval}(x + y \geq 0,\, p_2) = \bot$

Solving abduction with enumerative CEGIS

Consider T as LIA, Ax $= \{y \geq 0\}$, G $= \{x + y + z \geq 0\}$,

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P	Candidate
$\{p_1 = (0, 0, -1), p_2 = (-1, 0, 0)\}$	$x + y < 0$

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and let P be the set of points satisfying Ax and falsifying G.

P	Candidate
$\{p_1 = (0, 0, -1), p_2 = (-1, 0, 0)\}$	$x + y < 0$

$\textsc{eval}(x + y < 0,\, p_1) = \bot, \textsc{eval}(x + y < 0,\, p_2) = \top$

Solving abduction with enumerative CEGIS

Consider T as LIA, Ax $= \{y \geq 0\}$, G $= \{x + y + z \geq 0\}$,

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P	Candidate
$\{p_1 = (0, 0, -1), p_2 = (-1, 0, 0)\}$	$x + z \geq 0$

$\text{EVAL}(x + z \geq 0,\ p_1) = \perp, \text{EVAL}(x + z \geq 0,\ p_2) = \perp$

Solving abduction with enumerative CEGIS

Consider T as LIA, Ax $= \{y \geq 0\}$, G $= \{x + y + z \geq 0\}$,

$$R = \quad A \rightarrow 0 \mid 1 \mid x \mid y \mid z \mid A + A \qquad B \rightarrow A < A \mid A \geq A$$

and let P be the set of points satisfying Ax and falsifying G.

P	Candidate
$\{p_1 = (0, 0, -1), p_2 = (-1, 0, 0)\}$	$x + z \geq 0$ ✓

$x + z \geq 0 \wedge y \geq 0 \wedge x + y + z \not\geq 0$ is T-UNSAT and
$x + z \geq 0 \wedge y \geq 0$ is T-SAT

Scalability issues

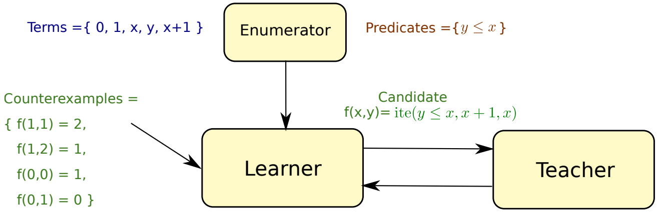
Enumerative CEGIS is effective but limited by the explosion of the enumeration space as term size increases

For this bit-vector grammar, enumerating

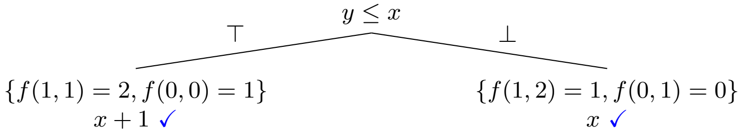
- Terms of size $= 1$: .05 seconds
- Terms of size $= 2$: .6 seconds
- Terms of size $= 3$: 48 seconds
- Terms of size $= 4$: 5.8 hours
- Terms of size $= 5$: ??? (100+ days)

```
(synth-fun f ((s (BitVec 4))
              (t (BitVec 4)))
 (BitVec 4) (
 (Start (BitVec 4) (
  s t #x0
  (bvneg  Start)
  (bvnot  Start)
  (bvadd  Start Start)
  (bvmul  Start Start)
  (bvand  Start Start)
  (bvlshr Start Start)
  (bvor   Start Start)
  (bvshl  Start Start))))
```

Divide and conquer SyGuS [ARU17; NSM18; BRLT19]



- Generate partial solutions correct on examples seen so far
- Unify partial solutions (e.g. via decision tree learning)



- D&C provides much better scalability

Scalable syntax-guided synthesis for abduction

- We extend the procedure by unifying partial solutions into conjunctions
- Besides P also maintains
  - a set E of enumerated formulas
- Candidates C are subsets of E such that
  - For every point in P at least one element of C is false
    Otherwise $\textsc{eval}(\mathsf{Ax}[\bar{p}] \wedge \mathsf{C}[\bar{p}] \wedge \neg \mathsf{G}[\bar{p}]) = \top$

Scalable syntax-guided synthesis for abduction

- We extend the procedure by unifying partial solutions into conjunctions
- Besides P also maintains
  - a set E of enumerated formulas
  - a set U of subsets of E which are inconsistent with Ax
- Candidates C are subsets of E such that
  - For every point in P at least one element of C is false

    Otherwise $\textsc{eval}(\mathsf{Ax}[\bar{p}] \wedge \mathsf{C}[\bar{p}] \wedge \neg \mathsf{G}[\bar{p}]) = \top$
  - no subset of C occurs in U

    Otherwise C is inconsistent with Ax
- Leverages unsat cores to improve eagerly discarding candidates

Consider again $\mathsf{Ax} = \{y \geq 0\}$ and $\mathsf{G} = \{x + y + z \geq 0\}$

$e \in \mathsf{E}$	U	$p \in \mathsf{P}$	C

Consider again Ax $= \{y \geq 0\}$ and G $= \{x + y + z \geq 0\}$

$e \in$ E	U	$p \in$ P	C
$x \geq 0$			

Consider again Ax $= \{y \geq 0\}$ and G $= \{x + y + z \geq 0\}$

$e \in$ E	U	$p \in$ P	C
$x \geq 0$			$\{x \geq 0\}$

Consider again Ax $= \{y \geq 0\}$ and G $= \{x + y + z \geq 0\}$

$e \in$ E	U	$p \in$ P	C
$x \geq 0$			$\{x \geq 0\}$

$x \geq 0 \wedge y \geq 0 \wedge x + y + z \not\geq 0$ is T-SAT, $p_1 = (0, 0, -1)$

Consider again Ax $= \{y \geq 0\}$ and G $= \{x + y + z \geq 0\}$

$e \in$ E	U	$p \in$ P	C
$x \geq 0$		$p_1 = (0, 0, -1)$	

Consider again Ax $= \{y \geq 0\}$ and G $= \{x + y + z \geq 0\}$

$e \in$ E	U	$p \in$ P	C
$x \geq 0$		$p_1 = (0, 0, -1)$	
$x < 0$			

Consider again $\mathsf{Ax} = \{y \geq 0\}$ and $\mathsf{G} = \{x + y + z \geq 0\}$

$e \in \mathsf{E}$	U	$p \in \mathsf{P}$	C
$x \geq 0$		$p_1 = (0, 0, -1)$	$\{x < 0\}$
$x < 0$			

Consider again Ax $= \{y \geq 0\}$ and G $= \{x + y + z \geq 0\}$

$e \in$ E	U	$p \in$ P	C
$x \geq 0$		$p_1 = (0, 0, -1)$	$\{x < 0\}$
$x < 0$			

$$\textsc{eval}(x < 0,\, p_1) = \perp$$

Consider again Ax $= \{y \geq 0\}$ and G $= \{x + y + z \geq 0\}$

$e \in$ E	U	$p \in$ P	C
$x \geq 0$		$p_1 = (0, 0, -1)$	$\{x < 0\}$
$x < 0$			

$x < 0 \wedge y \geq 0 \wedge x + y + z \ngeq 0$ is T-SAT, $p_2 = (-1, 0, 0)$

Consider again $\mathsf{Ax} = \{y \geq 0\}$ and $\mathsf{G} = \{x + y + z \geq 0\}$

$e \in \mathsf{E}$	U	$p \in \mathsf{P}$	C
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$e \in \mathsf{E}$	U	$p \in \mathsf{P}$	C
$x \geq 0$		$p_1 = (0, 0, -1)$	$\{x < 0, x \geq 0\}$
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Consider again Ax $= \{y \geq 0\}$ and G $= \{x + y + z \geq 0\}$

$e \in$ E	U	$p \in$ P	C
$x \geq 0$		$p_1 = (0, 0, -1)$	$\{x < 0, x \geq 0\}$
$x < 0$		$p_2 = (-1, 0, 0)$	

$\text{EVAL}(x < 0,\ p_1) = \perp, \text{EVAL}(x \geq 0,\ p_2) = \perp$

Consider again Ax $= \{y \geq 0\}$ and G $= \{x + y + z \geq 0\}$

$e \in$ E	U	$p \in$ P	C
$x \geq 0$		$p_1 = (0, 0, -1)$	$\{x < 0, x \geq 0\}$
$x < 0$		$p_2 = (-1, 0, 0)$	

$x < 0 \land x \geq 0 \land y \geq 0$ is T-unsat, $u = \{x < 0, x \geq 0\}$

Consider again $\mathsf{Ax} = \{y \geq 0\}$ and $\mathsf{G} = \{x + y + z \geq 0\}$

$e \in \mathsf{E}$	U	$p \in \mathsf{P}$	C
$x \geq 0$		$p_1 = (0, 0, -1)$	
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Consider again Ax $= \{y \geq 0\}$ and G $= \{x + y + z \geq 0\}$

$e \in$ E	U	$p \in$ P	C
$x \geq 0$		$p_1 = (0, 0, -1)$	
$x < 0$	$\{x < 0, x \geq 0\}$	$p_2 = (-1, 0, 0)$	
$y \geq 0$			

Consider again Ax $= \{y \geq 0\}$ and G $= \{x + y + z \geq 0\}$

$e \in$ E	U	$p \in$ P	C
$x \geq 0$		$p_1 = (0, 0, -1)$	$\{y \geq 0\}$
$x < 0$	$\{x < 0, x \geq 0\}$	$p_2 = (-1, 0, 0)$	
$y \geq 0$			

Consider again $\mathsf{Ax} = \{y \geq 0\}$ and $\mathsf{G} = \{x + y + z \geq 0\}$

$e \in \mathsf{E}$	U	$p \in \mathsf{P}$	C
$x \geq 0$		$p_1 = (0, 0, -1)$	$\{y \geq 0\}$
$x < 0$	$\{x < 0, x \geq 0\}$	$p_2 = (-1, 0, 0)$	
$y \geq 0$			

$\text{EVAL}(y \geq 0,\ p_1) = \top, \text{EVAL}(y \geq 0,\ p_2) = \top$

Consider again Ax $= \{y \geq 0\}$ and G $= \{x + y + z \geq 0\}$

$e \in$ E	U	$p \in$ P	C
$x \geq 0$		$p_1 = (0, 0, -1)$	
$x < 0$	$\{x < 0, x \geq 0\}$	$p_2 = (-1, 0, 0)$	
$y \geq 0$			
$y < 0$			

Consider again Ax $= \{y \geq 0\}$ and G $= \{x + y + z \geq 0\}$

$e \in$ E	U	$p \in$ P	C
$x \geq 0$		$p_1 = (0, 0, -1)$	$\{y < 0\}$
$x < 0$	$\{x < 0, x \geq 0\}$	$p_2 = (-1, 0, 0)$	
$y \geq 0$			
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Consider again Ax $= \{y \geq 0\}$ and G $= \{x + y + z \geq 0\}$

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$x \geq 0$		$p_1 = (0, 0, -1)$	$\{y < 0\}$
$x < 0$	$\{x < 0, x \geq 0\}$	$p_2 = (-1, 0, 0)$	
$y \geq 0$			
$y < 0$			

$\text{EVAL}(y < 0,\, p_1) = \perp, \text{EVAL}(y < 0,\, p_2) = \perp$

Consider again Ax $= \{y \geq 0\}$ and G $= \{x + y + z \geq 0\}$

$e \in$ E	U	$p \in$ P	C
$x \geq 0$		$p_1 = (0, 0, -1)$	$\{y < 0\}$
$x < 0$	$\{x < 0, x \geq 0\}$	$p_2 = (-1, 0, 0)$	
$y \geq 0$			
$y < 0$			

$y < 0 \land y \geq 0$ is T-unsat, $u = \{y < 0\}$

Consider again Ax $= \{y \geq 0\}$ and G $= \{x + y + z \geq 0\}$

$e \in$ E	U	$p \in$ P	C
$x \geq 0$		$p_1 = (0, 0, -1)$	
$x < 0$	$\{x < 0, x \geq 0\}$	$p_2 = (-1, 0, 0)$	
$y \geq 0$			
$y < 0$	$\{y < 0\}$		

Consider again $\text{Ax} = \{y \geq 0\}$ and $\text{G} = \{x + y + z \geq 0\}$

$e \in \text{E}$	U	$p \in \text{P}$	C
$x \geq 0$		$p_1 = (0, 0, -1)$	
$x < 0$	$\{x < 0, x \geq 0\}$	$p_2 = (-1, 0, 0)$	
$y \geq 0$			
$y < 0$	$\{y < 0\}$		
$z \geq 0$			

Consider again $\mathsf{Ax} = \{y \geq 0\}$ and $\mathsf{G} = \{x + y + z \geq 0\}$

$e \in \mathsf{E}$	U	$p \in \mathsf{P}$	C
$x \geq 0$		$p_1 = (0, 0, -1)$	$\{x \geq 0, z \geq 0\}$
$x < 0$	$\{x < 0, x \geq 0\}$	$p_2 = (-1, 0, 0)$	
$y \geq 0$			
$y < 0$	$\{y < 0\}$		
$z \geq 0$			

Consider again $\mathsf{Ax} = \{y \geq 0\}$ and $\mathsf{G} = \{x + y + z \geq 0\}$

$e \in \mathsf{E}$	U	$p \in \mathsf{P}$	C
$x \geq 0$		$p_1 = (0, 0, -1)$	$\{x \geq 0, z \geq 0\}$
$x < 0$	$\{x < 0, x \geq 0\}$	$p_2 = (-1, 0, 0)$	
$y \geq 0$			
$y < 0$	$\{y < 0\}$		
$z \geq 0$			

$\text{EVAL}(z \geq 0,\, p_1) = \bot, \text{EVAL}(x \geq 0,\, p_2) = \bot$

Consider again Ax $= \{y \geq 0\}$ and G $= \{x + y + z \geq 0\}$

$e \in$ E	U	$p \in$ P	C
$x \geq 0$		$p_1 = (0, 0, -1)$	$\{x \geq 0, z \geq 0\}$
$x < 0$	$\{x < 0, x \geq 0\}$	$p_2 = (-1, 0, 0)$	
$y \geq 0$			
$y < 0$	$\{y < 0\}$		
$z \geq 0$			

$x \geq 0 \wedge z \geq 0 \wedge y \geq 0 \wedge x + y + z \not\geq 0$ is T-UNSAT and
$x \geq 0 \wedge z \geq 0 \wedge y \geq 0$ is T-SAT

Consider again Ax $= \{y \geq 0\}$ and G $= \{x + y + z \geq 0\}$

$e \in$ E	U	$p \in$ P	C
$x \geq 0$		$p_1 = (0, 0, -1)$	$\{x \geq 0, z \geq 0\}$ ✓
$x < 0$	$\{x < 0, x \geq 0\}$	$p_2 = (-1, 0, 0)$	
$y \geq 0$			
$y < 0$	$\{y < 0\}$		
$z \geq 0$			

$x \geq 0 \wedge z \geq 0 \wedge y \geq 0 \wedge x + y + z \not\geq 0$ is T-UNSAT and
$x \geq 0 \wedge z \geq 0 \wedge y \geq 0$ is T-SAT

Incremental weakening

- Algorithms can be extended to incrementally generate weaker solutions

- Generate new abduct C and test if C $\wedge$ Ax $\wedge \neg S$ is T-satisfiable
  - If yes, then C has more models than S consistent with Ax
  - S is updated to S $\vee$ C

Evaluation

Setup

- Two configurations of CVC4SY

CVC4SY+B	abduction via enumerative CEGIS
CVC4SY+U	abduction via divide and conquer

- GPiD and EXPLAIN as baselines
- 300s timeout, 8gb RAM, Intel E5-2637 v4 CPUs, Ubuntu 16.04
- We had to generate our own benchmarks
  - No existing standard benchmark library for abduction
  - Integration into verification tools was beyond the scope of this work

Full data at `http://cvc4.cs.stanford.edu/papers/abduction-sygus/`

Benchmark selection

- Three relevant-to-verification SMT-LIB logics:
  - QF_LIA, QF_NIA, and QF_SLIA
- From SAT benchmarks $\varphi = \psi_1 \wedge \cdots \wedge \psi_n$:
  - $\underbrace{\psi_1 \wedge \cdots \wedge \psi_{n-1}}_{\text{Axioms}} \wedge \underbrace{\mathsf{S}}_{\text{Solution}} \models \underbrace{\neg\psi_n}_{\text{Goal}}$
- Grammars are generated based on logic and benchmark variables
- Example of grammar for QF_LIA problem with variables $x_1, \ldots, x_n$:

$$\begin{array}{lcl} A & \to & x_1 \mid \cdots \mid x_n \mid 0 \mid 1 \mid A + A \mid A - A \mid \mathsf{ite}(B,\ A,\ A) \\ B & \to & A \geq A \mid A \simeq A \mid \neg B \end{array}$$

Finding missing assumptions in SAT benchmarks

		cvc4sy+b		cvc4sy+u	
Logic	#	Solved	Unique	Solved	Unique
QF_SLIA	11954	10902	3	10980	81
QF_LIA	2025	721	261	594	134
QF_NIA	12214	1492	171	1712	391
Total	26593	13329	435	13628	606

- Can any solution be found in 300s?
  - Orthogonality in QF_{LIA,NIA} probably due to fragility of integer solvers

Finding missing assumptions in SAT benchmarks

		CVC4SY+B			CVC4SY+U		
Logic	#	Solved	Unique	Weaker	Solved	Unique	Weaker
QF_SLIA	11954	10902	3	466	10980	81	0
QF_LIA	2025	721	261	183	594	134	2
QF_NIA	12214	1492	171	671	1712	391	45
Total	26593	13329	435	1320	13628	606	47

- Can any solution be found in 300s?
  - Orthogonality in QF_{LIA,NIA} probably due to fragility of integer solvers
- Who finds weaker solution overall?
  - CVC4SY+U has better success rate but often produces stronger solutions

Comparison with Explain

- Restricted to QF_LIA as it was better supported by Explain
- Explain solves harder problem: solutions with minimal variable set

- Can any solution be found in 300s?

	Solved	Unique	Total time
cvc4sy+b	721	261	418849s
cvc4sy+u	594	125	449424s
Explain	33	0	532839s

Comparison with Explain

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- Explain solves harder problem: solutions with minimal variable set

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	Solved	Unique	Total time
cvc4sy+b	721	261	418849s
cvc4sy+u	594	125	449424s
Explain	33	0	532839s

- In incremental mode cvc4sy+u finds solution with minimal number of variables to 25 of the 33 Explain solves

Comparison with GPID

- Restricted to 400 satisfiable QF_UFLIA benchmarks used in [EPS18]
  - While GPID's method is theory agnostic, their tooling restricts usage
- GPID solves similar problem: implicates for satisfiable benchmarks
  - No axioms, goal is whole original formula
  - Uses pre-computed *abduces*. GPID-1 restricts abduces to size 1
- Can any solution be found in 300s?

	Solved	Unique	Total time
CVC4SY+B	214	0	57290s
CVC4SY+U	342	0	18735s
GPID	193	0	69s
GPID-1	398	54	1188s

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	Solved	Unique	Total time
CVC4SY+B	214	0	57290s
CVC4SY+U	342	0	18735s
GPID	193	0	69s
GPID-1	398	54	1188s

- GPID heavily dependent on pre-computed abduces
- CVC4SY+U 30% slower on average than GPID-1 on commonly solved

Conclusions

Conclusions

- New scalable enumerative SyGuS framework for abduction
  - General
  - Flexible

- Evaluation shows favorable comparison with abduction tools

- Future work:
  - Integration into verification engines
  - Generating conditional rewrite rules for SMT solvers
    - Synthesize most general condition under which two terms are equivalent
    - Generalizes semi-automated development of rewrite rules [NRB+19]
  - Lifting approach to interpolation

Scalable Algorithms for Abduction via Enumerative Syntax-Guided Synthesis

Andrew Reynolds, *Haniel Barbosa*, Daniel Larraz, Cesare Tinelli

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Completing UNSAT cores

- From SMT-LIB benchmarks with minimal unsat cores U:
  - $U \setminus \{\psi_{\mathsf{G}}, \psi_{\max}\} \wedge \mathsf{S} \models \neg\psi_{\mathsf{G}}$, with S as weak as $\psi_{\max}$, where
  - $\psi_{\max}$ is U's component with maximal size, used as a *reference*
  - $\psi_{\mathsf{G}} \in U \setminus \{\psi_{\max}\}$
- We chose the goal as the last formula in the core (viewed as a list) after the reference was removed
- We used Z3 to compute minimal unsat cores (120s)
- Excluded cores with less than three assertions

Completing UNSAT cores

- Can any solution at least as weak as the reference be found in 300s?

		CVC4SY+B		CVC4SY+U	
Logic	#	Solved	Unique	Solved	Unique
QF_LIA	97	6	0	6	0
QF_SLIA	2546	2546	32	2514	0
QF_NIA	781	86	49	41	4
Total	3424	2638	81	2561	4

Completing UNSAT cores

▷ Can any solution at least as weak as the reference be found in 300s?

		CVC4SY+B		CVC4SY+U	
Logic	#	Solved	Unique	Solved	Unique
QF_LIA	97	6	0	6	0
QF_SLIA	2546	2546	32	2514	0
QF_NIA	781	86	49	41	4
Total	3424	2638	81	2561	4

▷ CVC4SY+B significantly outperforms CVC4SY+U in QF_SLIA

- Small references (generally size < 3) void need for specialized procedure

▷ Overall CVC4SY+B has an advantage for finding weaker solutions

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