

Extending enumerative function synthesis via SMT-driven classification

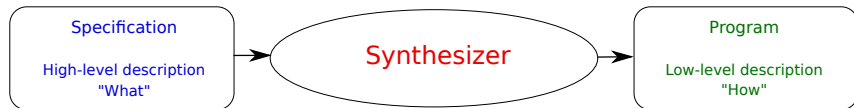
Haniel Barbosa, Andrew Reynolds, Daniel Larraz, Cesare Tinelli



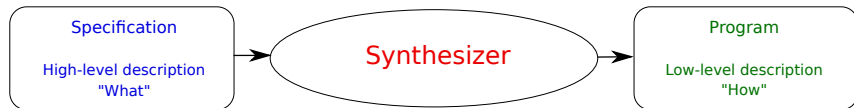
Lógicos em Quarentena

2020-04-30, The Internet

Program synthesis: “The Holy Grail of Computer Science”



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In	Out
0	1
1	2
2	4
3	8

$$P(x) =$$

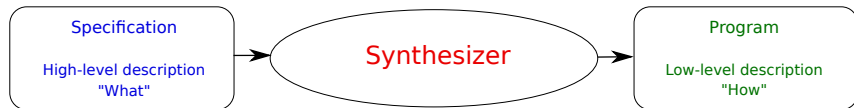
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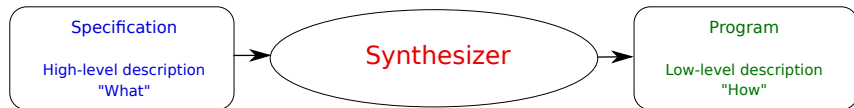


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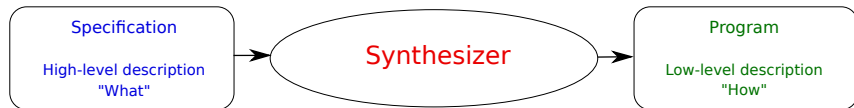
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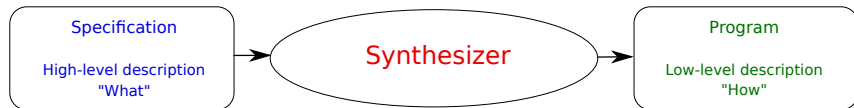
$$P(x) = \dots$$

Program synthesis: “The Holy Grail of Computer Science”



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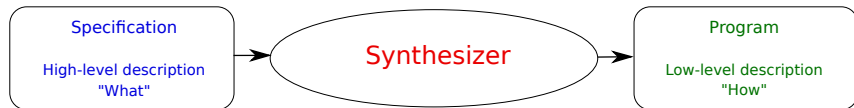
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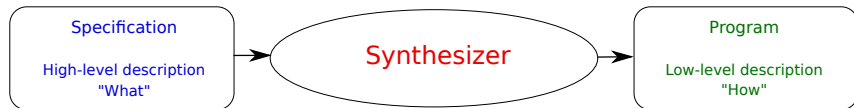


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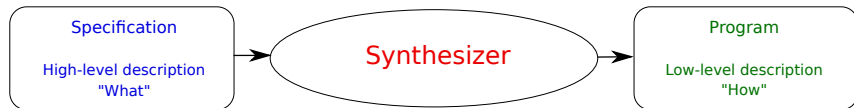
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Program synthesis: “The Holy Grail of Computer Science”



▷ Main challenges:

- ▶ Exploring search space
- ▶ Capturing intention

▷ Three main characteristics

- ▶ How to write specification
- ▶ How to constrain search space
- ▶ How to guide the search

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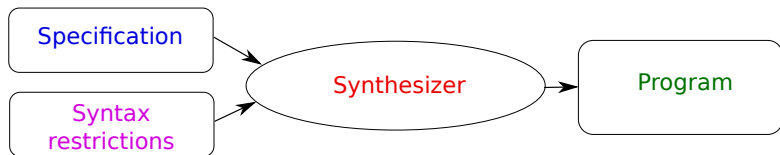
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Some applications of program synthesis

- ▷ Superoptimization [SSA13], [NRB+19], ...
- ▷ Program repair [NWK+17], [LCL+17], ...
- ▷ Programming by examples [Gu11], [FMG+17] ...
- ▷ Circuit synthesis [EWW16], ...
- ▷ Loop invariant synthesis [GLM+14], [PSM16], ...
- ▷ ...



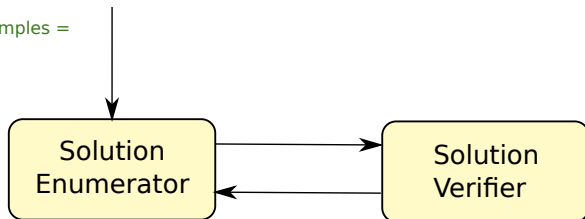
- ▷ Specification is given by (second-order) \mathcal{T} -formula: $\exists f. \forall \bar{x}. \varphi[f, \bar{x}]$
- ▷ Syntactic restrictions given by **context-free grammar** R

Consider the example:

$$\varphi = f(x, x) \simeq x + 1 \wedge f(x, x + 1) \simeq x$$

$$R = \begin{array}{l} A \rightarrow 0 \mid 1 \mid x \mid y \mid A + A \mid \text{ite}(B, A, A) \\ B \rightarrow A \leq A \mid \neg B \end{array}$$

Counterexamples =
{ }

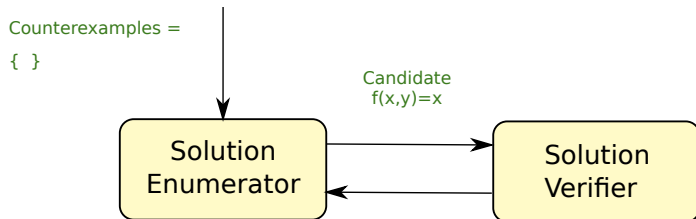


▷ De facto approach to SyGuS solving given its simplicity and efficacy

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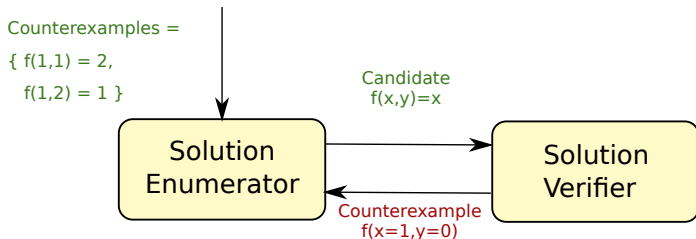


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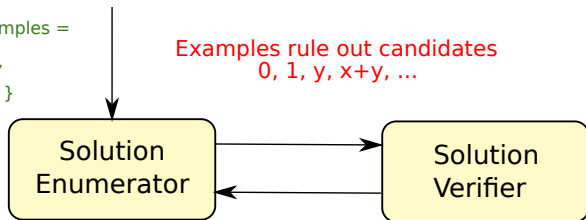
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Counterexamples =

{ f(1,1) = 2,
f(1,2) = 1 }

Examples rule out candidates
0, 1, y, x+y, ...

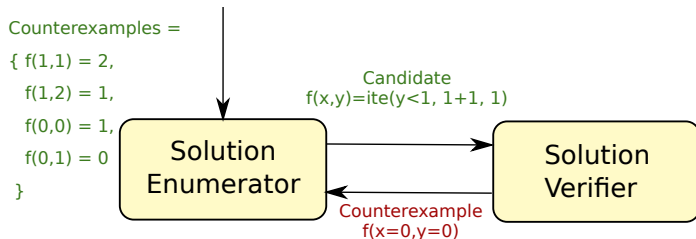


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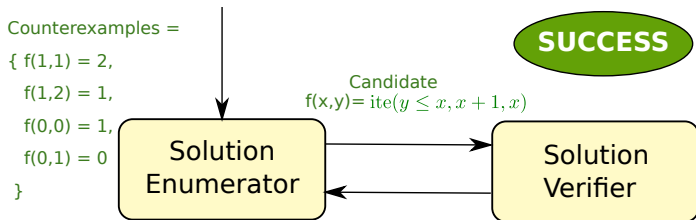


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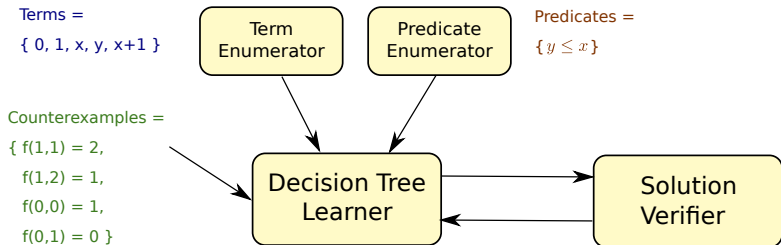
Scalability issues

Enumerative techniques are effective but limited by the explosion of the enumeration space as term size increases

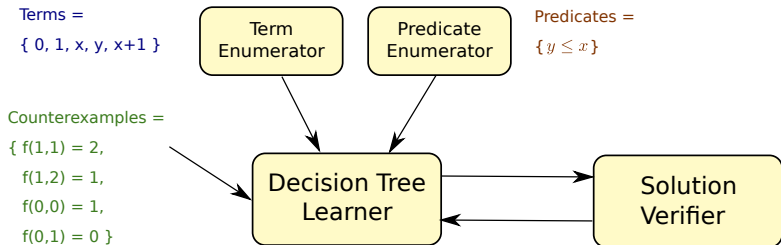
For this bit-vector grammar, enumerating

- ▶ Terms of size = 1 : .05 seconds
- ▶ Terms of size = 2 : .6 seconds
- ▶ Terms of size = 3 : 48 seconds
- ▶ Terms of size = 4 : 5.8 hours
- ▶ Terms of size = 5 : ??? (100+ days)

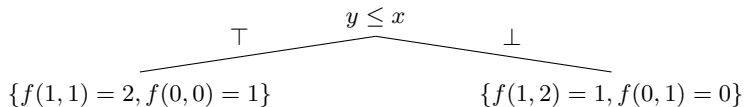
```
(synth-fun f ((s (BitVec 4))
              (t (BitVec 4))))
(BitVec 4) (
(Start (BitVec 4) (
s t #x0
(bvneg Start)
(bvnot Start)
(bvadd Start Start)
(bvmul Start Start)
(bvand Start Start)
(bvlshr Start Start)
(bvor Start Start)
(bvshl Start Start))))))
```

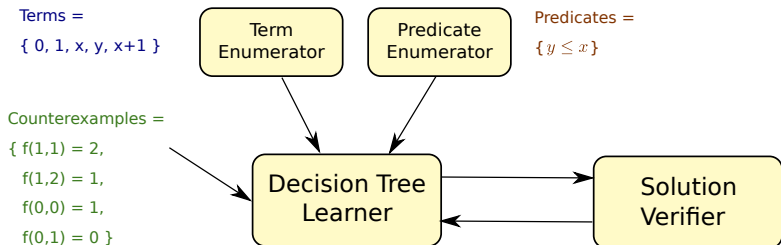


- ▷ Generate partial solutions correct on subset of input
- ▷ Unify partial solutions via decision tree learning

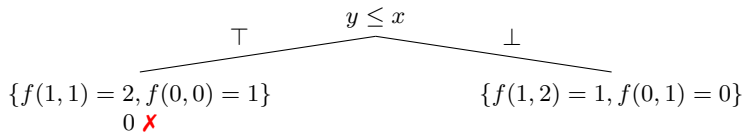


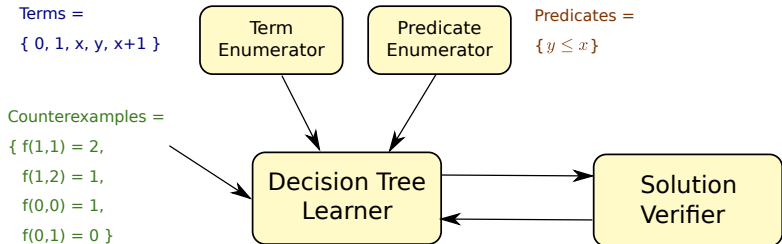
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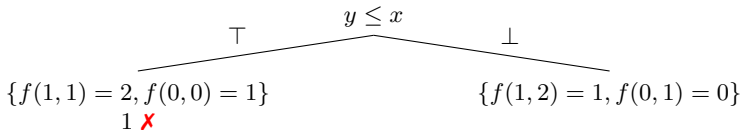


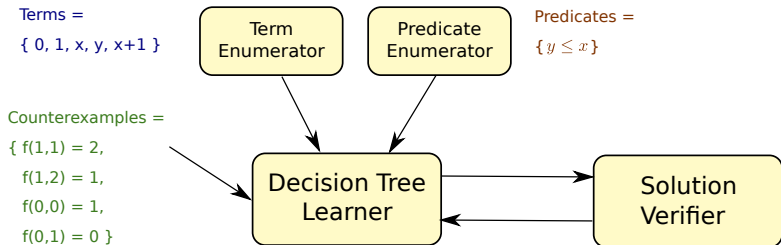
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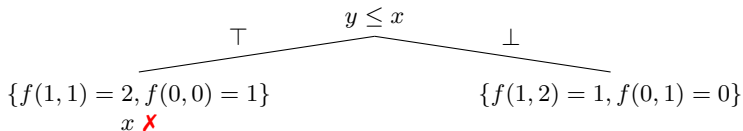


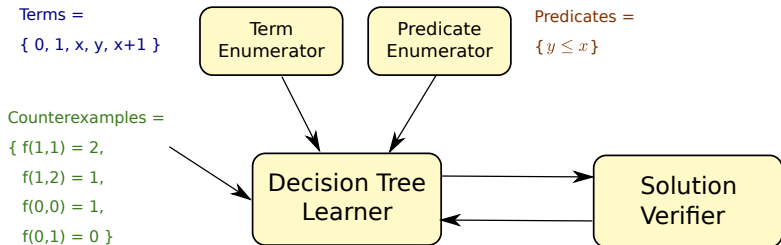
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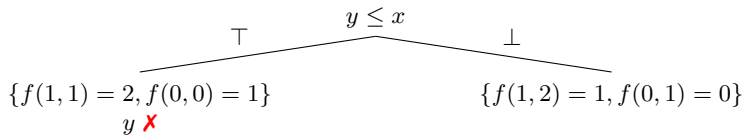


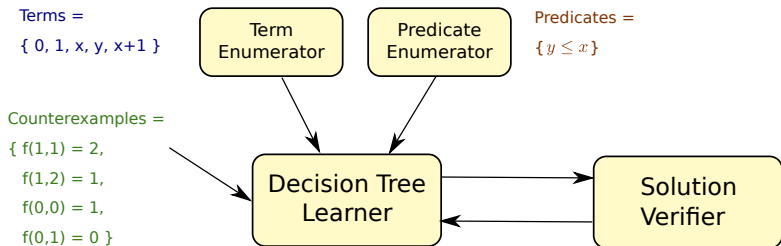
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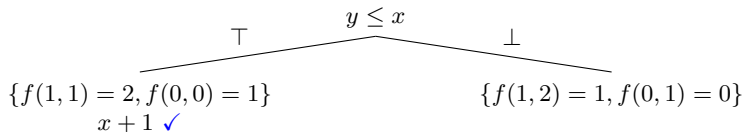


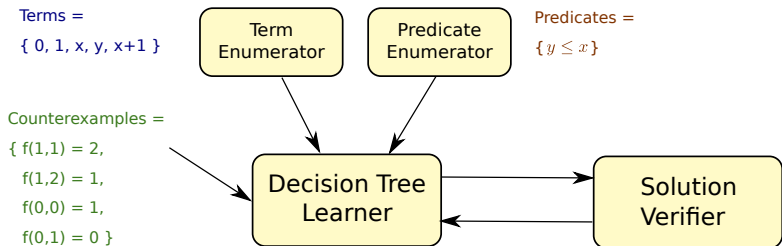
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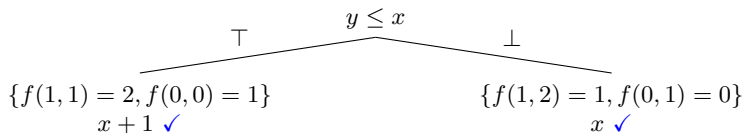


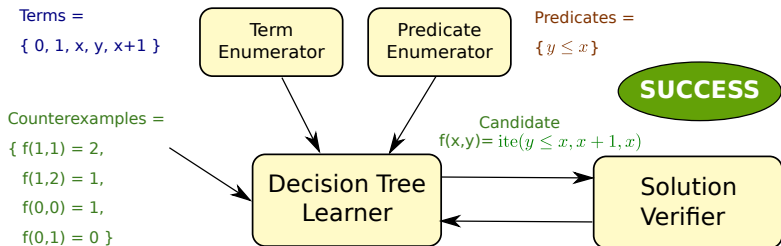
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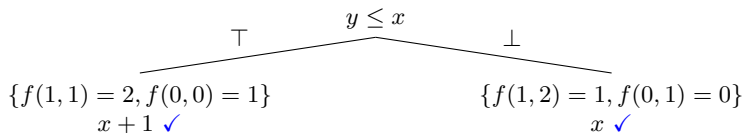


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- ▷ D&C provides much better scalability

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 - ▶ Each input valuation is specified independently

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Consider augmenting the previous example:

$$\varphi = \begin{array}{l} f(x, x) \simeq x + 1 \wedge f(x, x + 1) \simeq x \\ \wedge f(x, y) \simeq x + 1 \Rightarrow f(x + 2, y) \simeq x \end{array}$$

Counterexample $\{x \mapsto 1, y \mapsto 0\}$ yields the constraints:

$$f(1, 1) \simeq 2 \quad \wedge \quad f(1, 2) \simeq 1 \quad \wedge \quad f(1, 0) \simeq 2 \Rightarrow f(3, 0) \simeq 1$$

- ▷ A solution for $f(1, 0)$ restricts the solution for $f(3, 0)$
- ▷ Breaks assumption that partial solutions can be found *independently*

Challenges

- ▷ This limitation excludes interesting classes of synthesis problems
 - ▶ Invariants: $I(x) \wedge T(x, x') \Rightarrow I(x')$
 - ▶ Ranking functions: $rank(x') < rank(x)$
 - ▶ Modular arithmetic functions: $f(x) \simeq f(x + n)$
 - ▶ ...
- ▷ Extending D&C to arbitrary (non-point-wise) specifications:
 - ▶ Find a term assignment consistent with point dependencies

 - ▶ Correctly classify points according to term assignment

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Decision tree learning

- SMT-based solution-complete strategy
- Heuristic strategy

SMT solving for SyGuS

Satisfiability Modulo Theories (SMT)

First-order formulas

in CNF:

$$t ::= x \mid f(t, \dots, t)$$

$$\varphi ::= p(t, \dots, t) \mid \neg\varphi \mid \varphi \vee \varphi \mid \forall x_1 \dots x_n. \varphi$$

Given a formula φ in FOL and background theories $\mathcal{T}_1, \dots, \mathcal{T}_n$, finding a model \mathcal{M} giving an *interpretation* to all terms and predicates such that

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Example

Is φ satisfiable modulo *equality* and *arithmetic*?

$$\varphi = (x_1 \geq 0) \wedge (x_1 < 1) \quad \wedge \quad (f(x_1) \neq f(0)) \quad \vee \quad (x_3 + x_1 > x_3 + 1)$$

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$$\begin{array}{l} \varphi \models_{\text{LIA}} x_1 \simeq 0 \\ x_1 \simeq 0 \models_{\text{EUF}} f(x_1) \simeq f(0) \\ x_1 \simeq 0 \models_{\text{LIA}} x_3 + x_1 \not\simeq x_3 + 1 \end{array}$$

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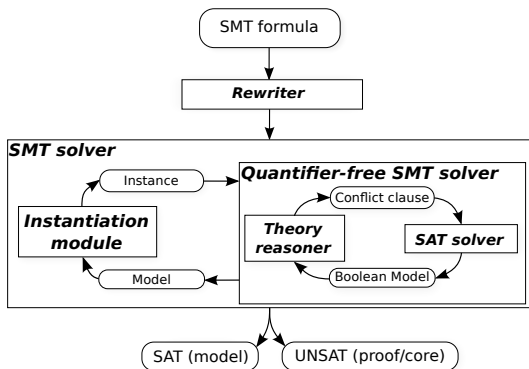
$$\varphi = \underbrace{(x_1 \geq 0) \wedge (x_1 < 1)}_{\text{LIA}} \wedge \underbrace{(f(x_1) \neq f(0))}_{\text{EUF}} \vee \underbrace{(x_3 + x_1 > x_3 + 1)}_{\text{LIA}}$$

$$\begin{array}{l} \varphi \models_{\text{LIA}} x_1 \simeq 0 \\ x_1 \simeq 0 \models_{\text{EUF}} f(x_1) \simeq f(0) \\ x_1 \simeq 0 \models_{\text{LIA}} x_3 + x_1 \not\simeq x_3 + 1 \end{array}$$

Therefore $\models_{\text{EUF} \cup \text{LIA}} \neg \varphi$

- ▷ Decidability depends on the theories being used
- ▷ Efficient decision procedures
 - ▶ Equality and uninterpreted functions ([Congruence Closure \(CC\)](#))
[NO80], [DST80]
 - ▶ Algebraic datatypes ([CC + Injectivity, Distinctness, Exhaustiveness, Acyclicity](#))
[BST07]
 - ▶ Linear arithmetic ([Simplex](#))
[DM06]
 - ▶ Bit-vectors ([Bit-blasting](#))
 - ▶ Combination of theories ([Nelson-Oppen](#))
 - ▶ ...
- ▷ Boolean search leverages SAT solvers
- ▷ Users may define their own theories
 - ▶ New operators as uninterpreted functions + Axioms

CDCL(\mathcal{T}) architecture



- ▷ Rewriter simplifies terms

$$x + 0 \rightarrow x \quad a \neq a \rightarrow \perp \quad (\text{str.replace } x \text{ (str.++ } x \text{) } y) \rightarrow x$$

- ▷ SAT solver enumerates models for Boolean skeleton of formula
- ▷ Theory solvers check consistency in the theory
- ▷ Instantiation module selects relevant instances

- ▷ Encode problem using a deep embedding into datatypes

$$\varphi = f(x, x) \simeq x + 1 \wedge f(x, x + 1) \simeq x$$

$$R = \begin{array}{l} A \rightarrow 0 \mid 1 \mid x \mid y \mid A + A \mid \text{ite}(B, A, A) \\ B \rightarrow A \leq A \mid \neg B \end{array}$$

Becomes

$$\llbracket \varphi \rrbracket = \text{eval}_a(d, x, x) \simeq x + 1 \wedge \text{eval}_a(d, x, x + 1) \simeq x$$

$$\llbracket R \rrbracket = \begin{array}{l} a = \text{Zero} \mid \text{One} \mid X \mid Y \mid \text{Plus}(a, a) \mid \text{Ite}(b, a, a) \\ b = \text{Leq}(a, a) \mid \text{Neg}(b) \end{array}$$

- ▷ eval maps datatype terms to their corresponding theory terms

- ▶ $\text{eval}_a(\text{Plus}(X, X), 2, 3)$ is interpreted as $(x + x)\{x \mapsto 2, y \mapsto 3\} = 4$

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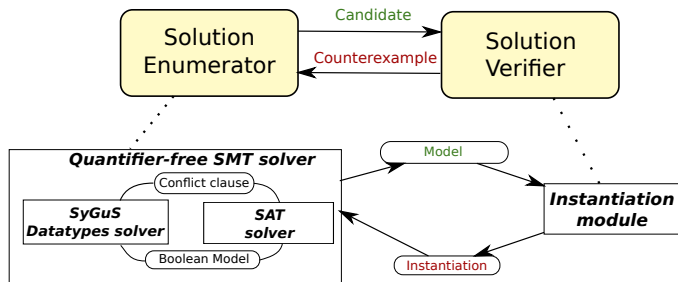
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- ▷ A solution is a model in which e.g.

- ▶ $d = \text{Ite}(\text{Leq}(Y, X), \text{Plus}(X, \text{One}), X)$, corresponding to
- ▶ $f = \lambda xy. \text{ite}(y \leq x, x + 1, x)$



- ▷ An instantiation module checks candidates against the specification
 - ▶ Generates lemmas witnessing why a candidate failed
- ▷ A specialized datatypes solver for SyGuS generates candidate solutions
 - ▶ Must satisfy all lemmas
 - ▶ Dedicated pruning
 - ▶ Parameterizable fairness criteria for enumeration

Unif+PI: a general divide-and-conquer framework for
SyGuS solving

Recapping

- ▷ D&C can only be applied to point-wise specifications
 - ▶ Each input valuation is specified independently
- ▷ Extending D&C to arbitrary (non-point-wise) specifications requires:
 - ▶ Find a term assignment consistent with point dependencies

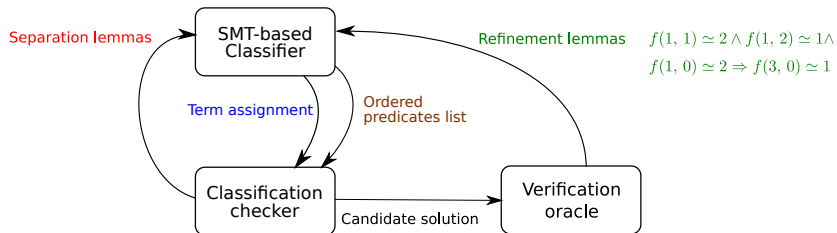
SMT solving

- ▶ Correctly classify points according to term assignment

Decision tree learning

- SMT-based solution-complete strategy
- Heuristic strategy

Unif+PI: Synthesis via Pointwise-Independent unification



▷ SMT-based classifier

- ▶ Assigns terms to points so that lemmas hold

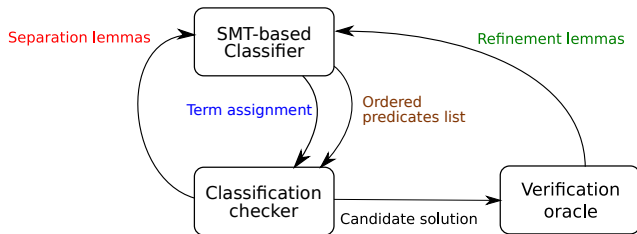
$$f(1, 1) \mapsto y + y, \quad \{f(1, 0), f(3, 0), f(1, 2)\} \mapsto x$$

- ▶ Generates ordered list of predicates to *separate* points: $P_1 \mapsto x \neq y$

▷ Classification checker: whether corresponding decision tree correctly classifies sample

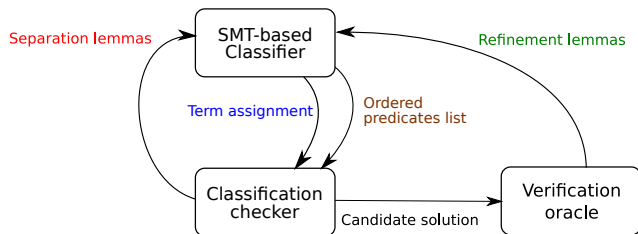
- ▶ Failures are encoded as *separation lemmas*

Unif+PI: Synthesis via Pointwise-Independent unification



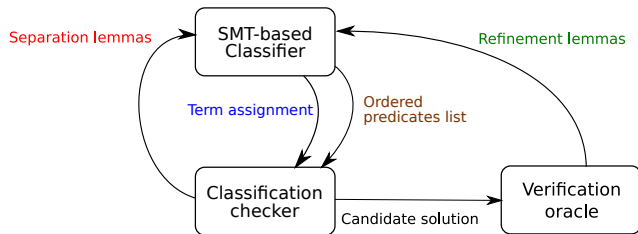
- ▷ Successful candidates that are not verified lead to refinement lemmas and the learning restarts

Unif+PI: Synthesis via Pointwise-Independent unification



- ▷ Successful candidates that are not verified lead to refinement lemmas and the learning restarts
- ▷ Bounded *solution-completeness* and *minimality* results due to exhaustive enumeration of possible classifiers according to
 - ▶ *size* and *number* of distinct terms to be assigned
 - ▶ *size* and *number* of distinct predicates

Unif+PI: Synthesis via Pointwise-Independent unification



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- ▷ Bounded *solution-completeness* and *minimality* results due to exhaustive enumeration of possible classifiers according to
 - ▶ *size* and *number* of distinct terms to be assigned
 - ▶ *size* and *number* of distinct predicates
- ▷ Our fairness criteria are $size = \log_2(\#terms)$, $\#pred = \#terms - 1$

Consider again:

$$\varphi = \begin{array}{l} f(x, x) \simeq x + 1 \wedge f(x, x + 1) \simeq x \\ \wedge f(x, y) \simeq x + 1 \Rightarrow f(x + 2, y) \simeq x \end{array}$$

▷ Initially a single term of size 0 will be a trivial successful classifier

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▷ Refinement lemma:

$$f(1, 1) \simeq 2 \quad \wedge \quad f(1, 0) \simeq 2 \Rightarrow f(3, 0) \simeq 1 \quad \wedge \quad f(1, 2) \simeq 1$$

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▷ This classifier fails on the sample, yielding a separation lemma

$$P_1 \simeq \top \Rightarrow f(1, 1) \simeq f(1, 0)$$

$$\begin{aligned} \varphi_R &= f(1, 1) \simeq 2 \quad \wedge \quad f(1, 0) \simeq 2 \Rightarrow f(3, 0) \simeq 1 \quad \wedge \quad f(1, 2) \simeq 1 \\ \varphi_S &= P_1 \simeq \top \Rightarrow f(1, 1) \simeq f(1, 0) \end{aligned}$$

- ▷ Given these constraints and current threshold the next candidate classifier produced is:

$$\begin{aligned} \{f(1, 1), f(1, 0), f(3, 0)\} &\mapsto y + 1, \quad f(1, 2) \mapsto 1 \\ P_1 &\mapsto y \leq x \end{aligned}$$

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- ▷ Running the classification checker:

$$f(1, 1)$$

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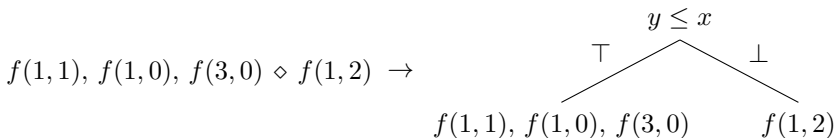
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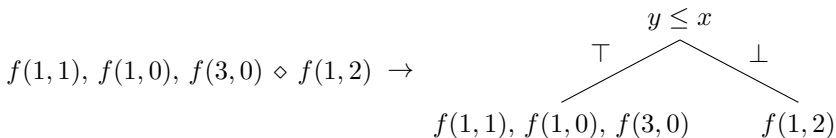


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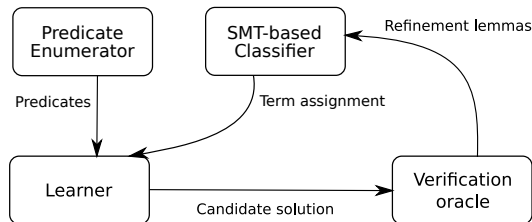
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- ▷ Running the classification checker:



- ▷ As the classification succeeds, a candidate is generated
- ▷ The candidate fails, so the process restarts with new refinement lemmas
- ▷ Eventually finds solution $f = \lambda xy. \text{ite}(x \leq y, \text{ite}(y \leq x, x + 1, x), y)$

Unif+PI with unconstrained predicate enumeration



- ▷ Unif+PI+E uses SMT solver only to produce term assignments
 - ▶ Relies on standard decision tree learning to classify a labeled sample
 - ▶ Predicates chosen from enumerated pool with information-gain heuristic
 - ▶ Separation conflicts solved when new predicates are enumerated
- ▷ Often sacrificing completeness and minimality allows problems to be solved more efficiently

Experimental results

Setup

- ▷ Benchmarks (all over LIA)
 - ▶ 127 invariant synthesis benchmarks from SyGuS-COMP'18
 - ▶ 440 invariant synthesis benchmarks from test suite of Kind 2
- ▷ Three configurations of CVC4SY

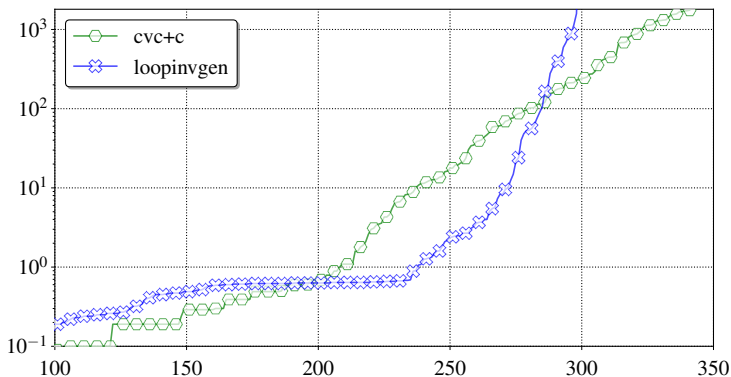
CVC+C	enumerative CEGIS [RBN+19]
CVC+UPI	Unif+PI
CVC+UPI+E	Unif+PI+E

- ▷ LOOPINVGEN [PM17] and CVC+C as baselines
- ▷ 1800s timeout, 8gb RAM

Full data at <http://cvc4.cs.stanford.edu/papers/FMCAD2019-UnifPI/>

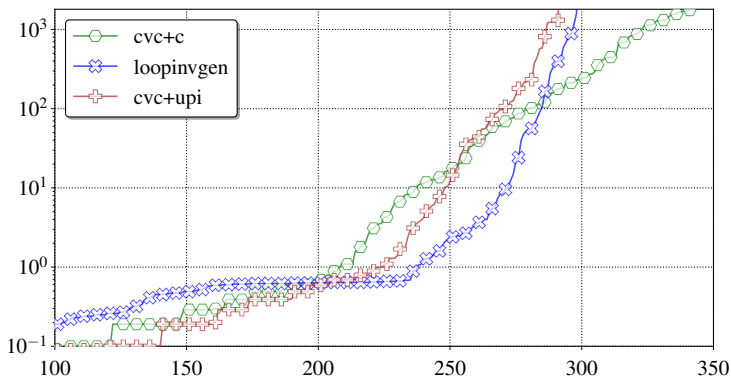
Summary

	Solved	Unique	Total time	Fastest	Shortest
CVC+C	341	30	436251s	245	259
LOOPINVGEN	298	7	433273s	261	289



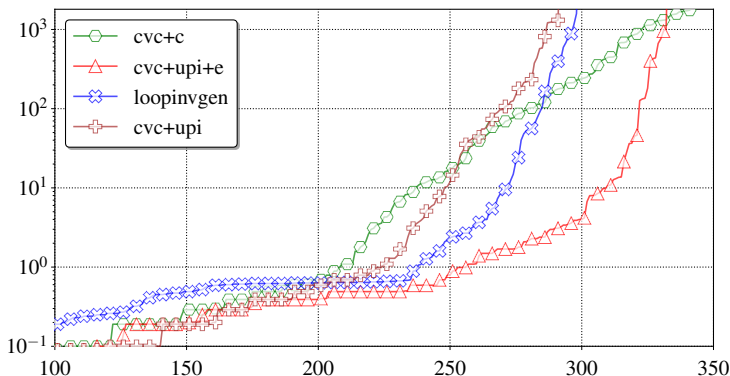
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Summary

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CVC+UPI+E	332	47	414356s	306	222
CVC+UPI	291	3	494534s	236	231
LOOPINVGEN	298	7	433273s	261	289
CVC-PORT	400	-	31476s	379	306



Advantages and disadvantages of Unif+PI

- ▷ CVC+UPI and CVC+UPI+E thrive when invariants can be built from combination of small literals
- ▷ CVC+C is superior when invariant is a single complex literal
 - ▶ 29 of its 30 unique solves are such cases
- ▷ CVC+UPI and CVC+UPI+E also suffer from dependence on samples
 - ▶ Sometimes search is biased towards simple classifiers when only a more complex one would suffice

Inv Track (829)

Solver	Solved	Fastest	Smallest	Score
CVC4-su	592	423	264	4493
LoopInvGen	512	442	364	4250
LoopInvGen-gplearn	511	411	349	4137
CVC4-Fast	522	319	243	3810
CVC4-Smart	539	283	260	3804
OASIS	538	20	317	3067
DryadSynth	277	161	39	1907



- ▷ 829 benchmarks from the literature in loop invariant synthesis
- ▷ 3600s timeout

Injecting some welcome realism

- ▷ Kind 2 employs in cooperation:
 - ▶ IC3 [Bra11]
 - ▶ k -induction [SSS00]
 - ▶ Generation of auxiliary invariants [KGT11]
- ▷ Kind 2 solves all the 480 benchmarks in its test suite in less than 120s
- ▷ Considering k -induction in isolation, CVC-PORT is competitive

	Solved	Unique	Time (commonly solved)
CVC-PORT	323	82	109.6
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- ▷ We consider this encouraging given our framework is
 - ▶ not theory-specific
 - ▶ single-threaded
 - ▶ not optimized for reachability

Conclusions

Conclusions

- ▷ New enumerative function synthesis framework via divide and conquer
 - ▶ No dependence on point-wise specifications
 - ▶ Powered by SMT-driven classification algorithms
 - ▶ Implemented in *CVC4SY*

- ▷ Experimental evaluation shows significant gains w.r.t. previous SyGuS techniques for invariant synthesis

Future work

▷ Improving classification

- ▶ Using constraint solving for synthesizing term assignments
- ▶ Only considering relevant arguments when synthesizing predicates

$$f(0, 0, 0, 1, 2, 1, 0) \diamond f(1, 0, 0, 5, 2, 1, 3)$$

- Can drastically reduce search space

▷ Improving sample

- ▶ Reducing noise: make points as similar as possible

$$f(1, 0, 0, 1, 2, 1, 0) \diamond f(1, 0, 0, 5, 2, 1, 0)$$

- ▶ Improve diversity via clustering analysis: only add new points to sample that are sufficiently different

Extending enumerative function synthesis via SMT-driven classification

Haniel Barbosa, Andrew Reynolds, Daniel Larraz, Cesare Tinelli



Lógicos em Quarentena

2020-04-30, The Internet

Extra slides

Invariant Synthesis

```
Add(Int x, y) {  
  z := x; i := 0;  
  assume(y > 0);  
  while (i < y) {  
    z := z + 1;  
    i := i + 1;  
  }  
  return z;  
}
```

Post-condition:

$\forall x, y : z = x + y$

Result is the sum
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Invariant?

Post-condition:

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Verification:

$z = x \wedge i = 0 \wedge y > 0$	\rightarrow	$Inv(x, y, z, i)$
$Inv(x, y, z, i) \wedge i < y \wedge z' = z + 1 \wedge i' = i + 1$	\rightarrow	$Inv(x, y, z', i')$
$Inv(x, y, z, i) \wedge i \geq y$	\rightarrow	$z = x + y$

Invariant Synthesis

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```

$Inv(x, y, z, i)$
 $z = x + i$
 $z \leq x + y$

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$\forall x, y : z = x + y$

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$Inv(x, y, z, i) \wedge i < y \wedge z' = z + 1 \wedge i' = i + 1$	\rightarrow	$Inv(x, y, z', i')$
$Inv(x, y, z, i) \wedge i \geq y$	\rightarrow	$z = x + y$

Invariant Synthesis in SyGuS

- ▷ State-of-the-art: LoopInvGen [PM17]: *data-driven* loop invariant inference with automatic feature synthesis
 - ▶ Precondition inference from sets of “good” and “bad” states
 - Feature synthesis for solving conflicts
 - ▶ PAC (*probably approximately correct*) algorithm for building candidate invariants

- ▷ “Bad” states are dependent on model of initial condition (no guaranteed convergence)

- ▷ No support for implication counterexamples

Invariant Synthesis with Unif+PI

- ▷ Refinement lemmas allows derivation of three kinds on data points:
 - ▶ “good points” (invariant must always hold)
 - ▶ “bad points” (invariant can never hold)
 - ▶ “implication points” (if invariant holds in first point it must hold in second)

- ▷ Native support for implication counterexamples

- ▷ Straightforward usage of classic information gain heuristic to build candidate solutions with decision tree learning
 - ▶ SMT solver “resolves” implication counterexample points as “good” and “bad”

 - ▶ Out-of-the-box Shannon entropy

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